UNIVERSITY OF TWENTE.

Formal Methods & Tools.

Symbolic reductions of probabilistic models using linear process equations

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5^{th} QUASIMODO meeting

Joint work with Joost-Pieter Katoen, Jaco van de Pol, and Mariëlle Stoelinga

Contents

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Probabilistic Model Checking

Probabilistic model checking:

- Verifying quantitative properties,
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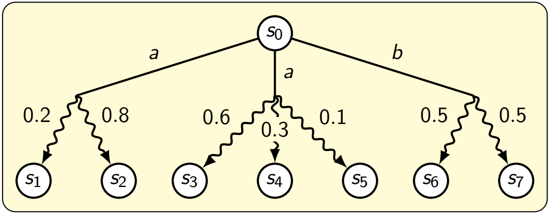

- Non-deterministically choose one of the three transitions
- Probabilistically choose the next state

Applications:

- Dependability analysis
- Performance analysis

Limitations and Solutions

Limitations of previous approaches:

- Susceptible to the state space explosion problem
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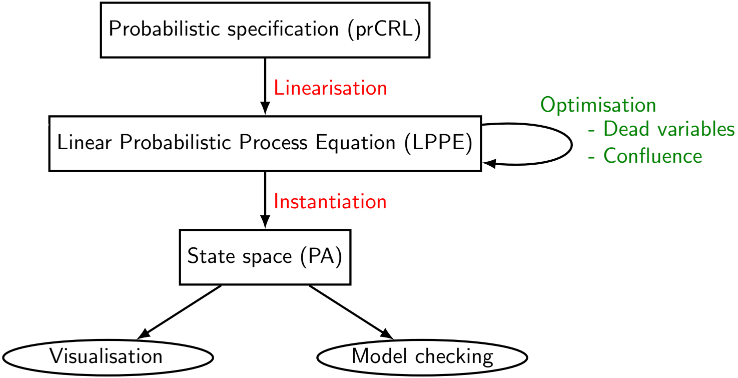

Overview of our approach

Our approach:

1. Specify systems in prCRL: a probabilistic process algebra incorporating both data types and probabilistic choice.
2. Transform specifications to LPPEs: a linear format enabling symbolic optimisations at the language level
3. Reduce state spaces before they are generated by manipulations of the linear format.

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A process algebra with data and probability: prCRL

Specification language prCRL:

- Based on μCRL (so data), with additional probabilistic choice
- Semantics defined in terms of probabilistic automata
- Minimal set of operators to facilitate formal manipulation
- Syntactic sugar easily definable

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The grammar of prCRL process terms

Process terms in prCRL are obtained by the following grammar:

$$p ::= Y(\vec{t}) \mid c \Rightarrow p \mid p + p \mid \sum_{x:D} p \mid a(\vec{t}) \sum\!\!\!\!\bullet_{x:D} f : p$$

Process equations and processes

A process equation is something of the form $X(\vec{g} : \vec{G}) = p$.

An example specification

Sending an arbitrary natural number

$$
\begin{aligned}
X(\text{active} : \text{Bool}) = {} & \\
& \text{not}(\text{active}) \Rightarrow \text{ping} \cdot \sum_{b:\text{Bool}} X(b) \\
+ \text{active} \quad & \Rightarrow \tau \sum\!\!\!\bullet_{n:\mathbb{N}^{>0}} \frac{1}{2^n} : \left(\text{send}(n) \cdot X(\text{false})\right)
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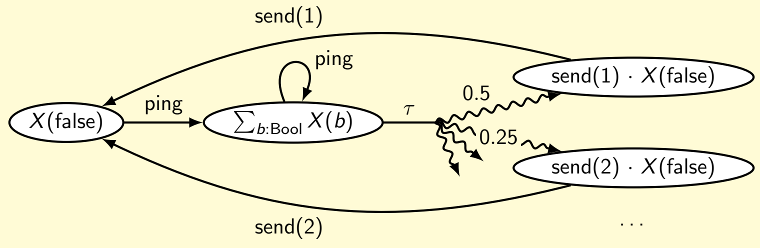

Compositionality using extended prCRL

For compositionality we introduce extended prCRL. It extends prCRL by parallel composition, encapsulation, hiding and renaming.

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$$X(n : \{1,2\}) = \text{write}_X(n) \cdot X(n) + \text{choose} \sum_{n' : \{1,2\}}^{\bullet} \frac{1}{2} : X(n')$$

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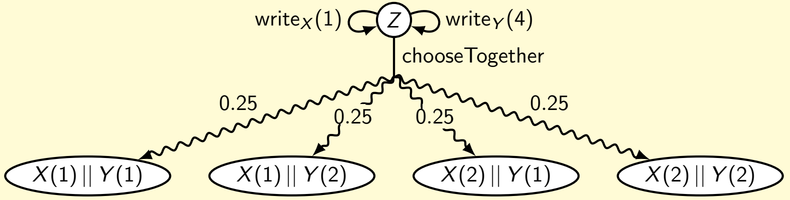

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A linear format for prCRL: the LPPE

LPPEs are a subset of prCRL specifications:

$$
\begin{aligned}
X(\vec{g} : \vec{G}) = & \sum_{\vec{d_1}:\vec{D_1}} c_1 \Rightarrow a_1(b_1) \sum\!\!\!\bullet_{\vec{e_1}:\vec{E_1}} f_1 : X(\vec{n_1}) \\
& \dots \\
+ & \sum_{\vec{d_k}:\vec{D_k}} c_k \Rightarrow a_k(b_k) \sum\!\!\!\bullet_{\vec{e_k}:\vec{E_k}} f_k : X(\vec{n_k})
\end{aligned}
$$

- $\vec{G}$ is a type for state vectors
- $\vec{D_i}$ a type for local variable vectors for summand i
- c_i is the enabling condition of summand i
- a_i is an atomic action, with action-parameter vector b_i
- $\vec{n_i}$ is the next-state vector of summand i.
- $\vec{E_i}$ a type for the probabilistic variable for summand i
- f_i is the probability distribution of summand i

Advantages of LPPEs

Advantages of using LPPEs instead of prCRL specifications:

- Easy state space generation
- Straight-forward parallel composition
- **Symbolic optimisations enabled at the language level**

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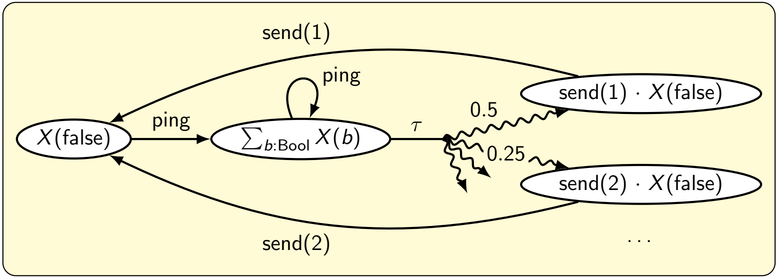
Theorem

Every specification S (without unguarded recursion) can be linearised to an LPPE S' in such a way that S and S' are strongly probabilistic bisimilar.

Linear probabilistic process equations – An example



Specification in prCRL

$$X(\text{active} : \text{Bool}) =$$

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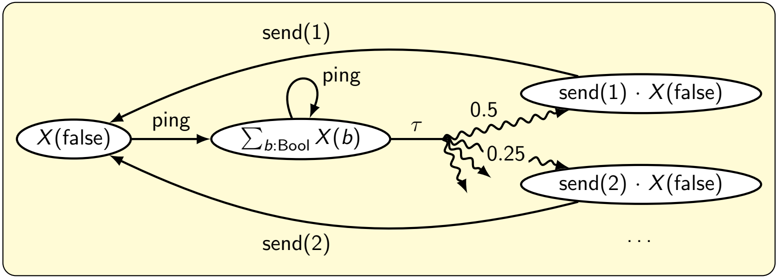

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Specification in LPPE

$$X(pc : \{1..3\}, n : \mathbb{N}^{\geq 0}) =$$
$$+ \; pc = 1 \Rightarrow \text{ping} \cdot X(2, 1)$$
$$+ \; pc = 2 \Rightarrow \text{ping} \cdot X(2, 1)$$
$$+ \; pc = 2 \Rightarrow \tau \sum_{n:\mathbb{N}^{>0}} \bullet \; \frac{1}{2^n} : X(3, n)$$
$$+ \; pc = 3 \Rightarrow \text{send}(n) \cdot X(1, 1)$$

Contents

Case study: a leader election protocol

- Implementation in Haskell:
  - Linearisation: from prCRL to LPPE
  - Parallel composition of LPPEs, hiding, renaming, encapsulation
  - Generation of the state space of an LPPE
  - Automatic constant elimination and summand simplification
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Leader election protocol à la Itai-Rodeh

- Two processes throw a die
  - *One of them throws a 6 $\rightarrow$ this will be the leader*
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- Two processes throw a die
  - *One of them throws a 6 → this will be the leader*
  - *Both throw 6 or neither throws 6 → throw again*
- More precise:
  - *Passive thread: receive value of opponent*
  - *Active thread: roll, send, compare (or block)*

A prCRL model of the leader election protocol

$$P(id : \{1, 2\}, val : Die, set : Bool) =$$

A prCRL model of the leader election protocol

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\begin{aligned}
&P(id : \{1,2\}, val : Die, set : Bool) = \\
&\quad set = false \Rightarrow \sum_{d:Die} rec(id, other(id), d) \cdot P(id, d, true))
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Reductions on the leader election protocol model

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$$\sum_{e21:D} pc21 = 3 \wedge pc11 = 1 \wedge set11 \wedge val11 = e21 \Rightarrow$$
$$checkVal(val11) \sum_{(k1,k2):\{*\}\times\{*\}}^{\bullet} multiply(1.0, 1.0):$$
$$Z(1, id11, val11, false, 1, 4, id21, d21, e21,$$
$$pc12, id12, val12, set12, d12, pc22, id22, d22, e22)$$

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Before reductions:

- 18 parameters
- 14 summands
- 3423 states
- 5478 transitions

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- 1613 states (-53%)
- 2278 transitions (-58%)

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Intuition:

$$\begin{tikzcd} \bullet \arrow[r, "\tau"] \arrow[d, "a"'] & \bullet \arrow[d, "a"] \\ \bullet \arrow[r, "\tau"] & \bullet \end{tikzcd}$$

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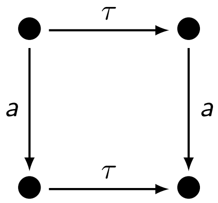

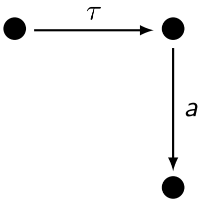

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Confluence reduction: efficiëntly reducing specifications while preserving branching probabilistic bisimulation.

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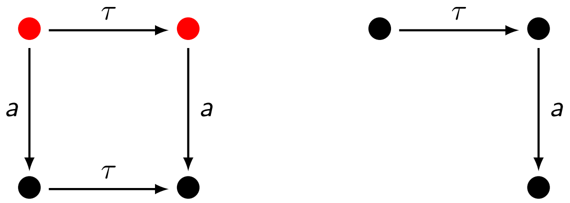

Overview of confluence reduction

Strong probabilistic bisimulation is sometimes too restrictive.

Confluence reduction: efficiëntly reducing specifications while preserving branching probabilistic bisimulation.

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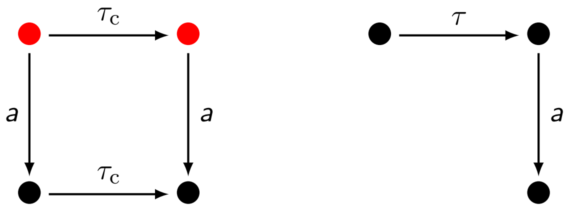

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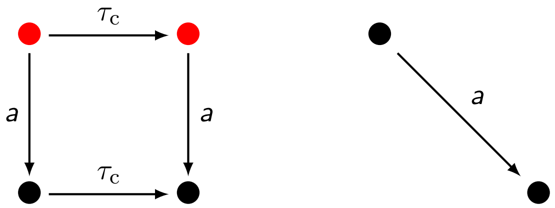
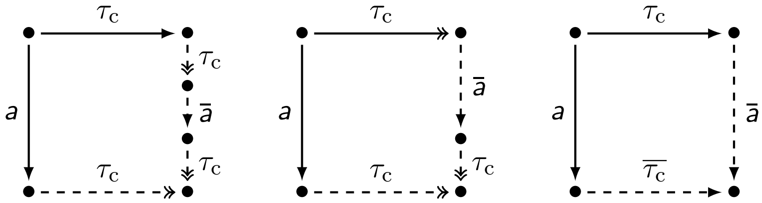
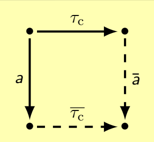


Variants of confluence

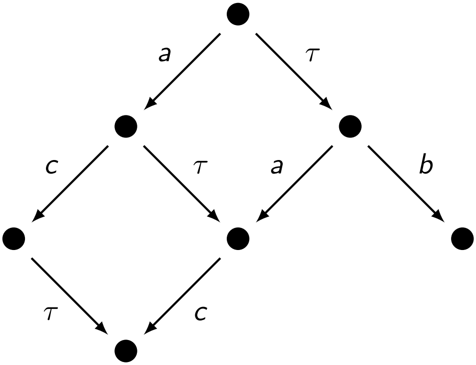


Weak confluence Confluence Strong confluence

Examples



Reduction based on strong confluence



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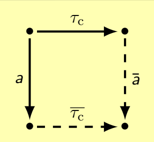

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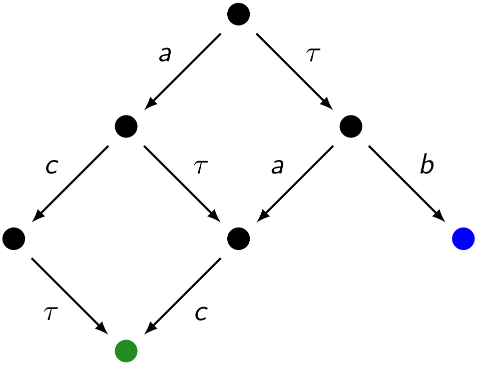

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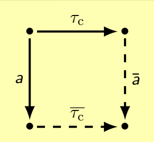

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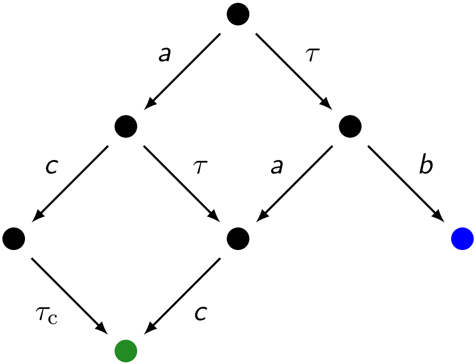

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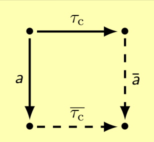

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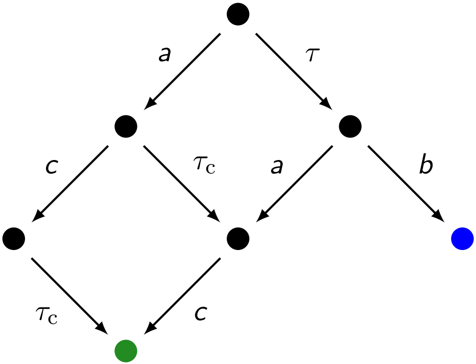

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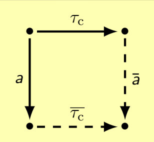

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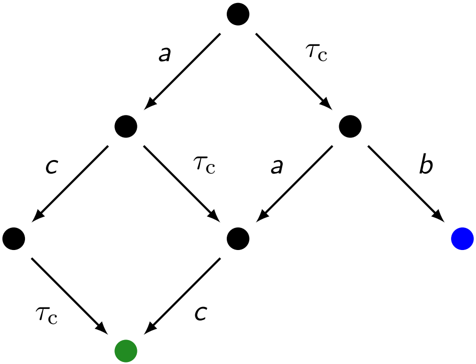

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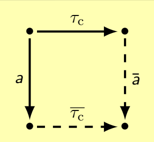

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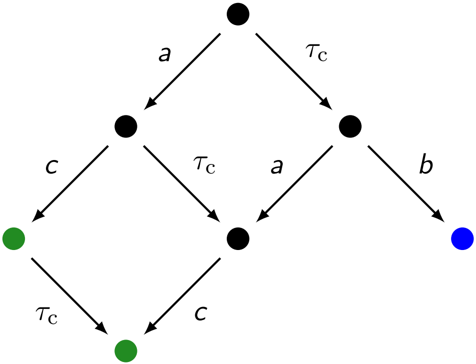

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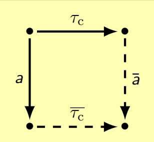

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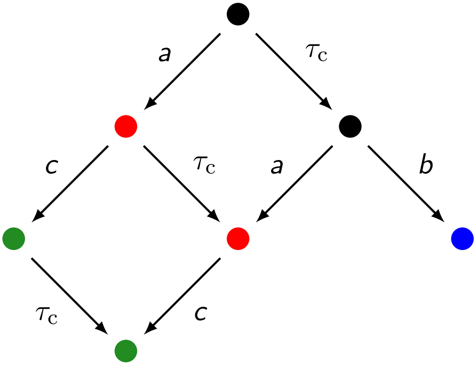

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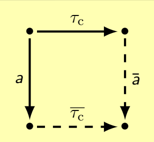

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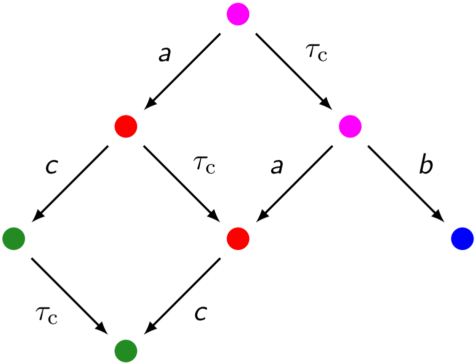
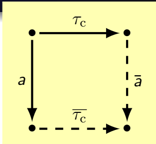


Examples



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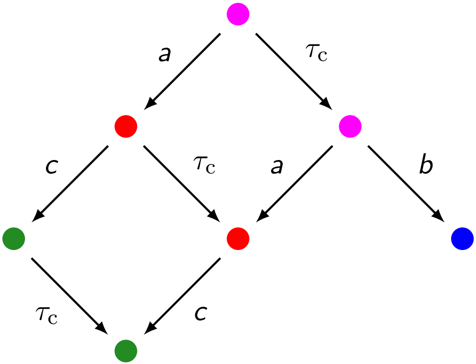

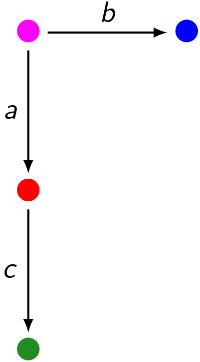

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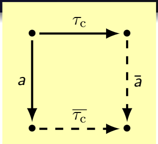

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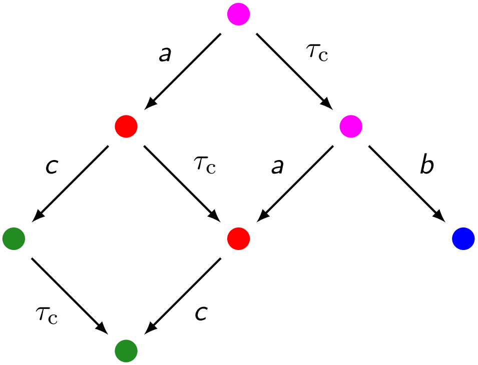
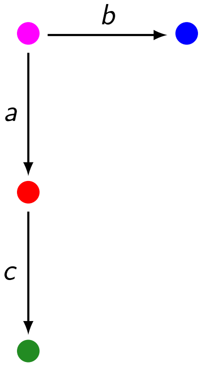
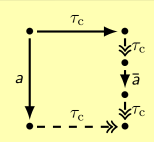


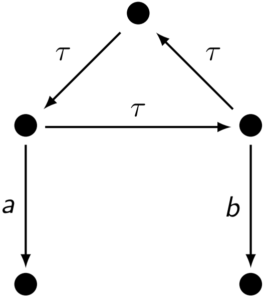


Giving τ_c steps priority works because of the absence of τ_c loops.

Examples



Reduction based on weak confluence



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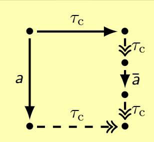

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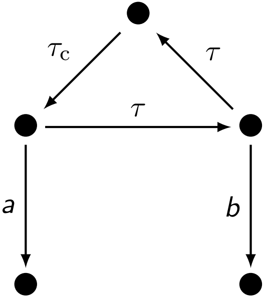

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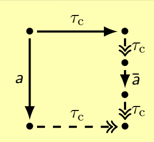

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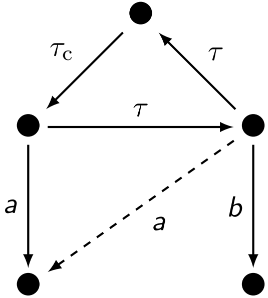

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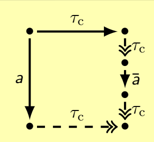

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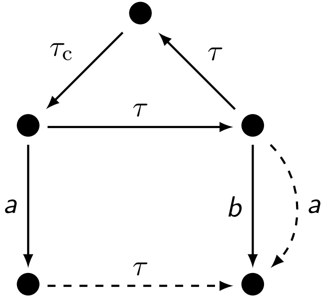

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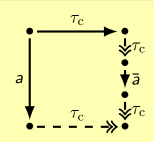

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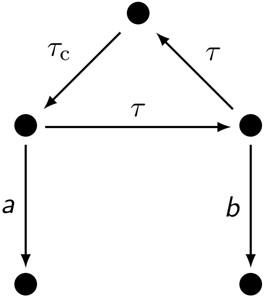

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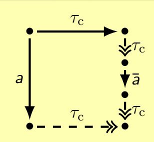

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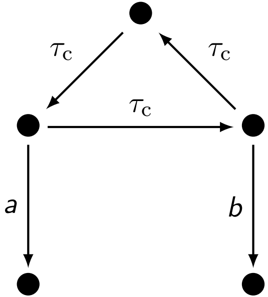

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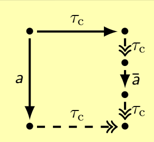

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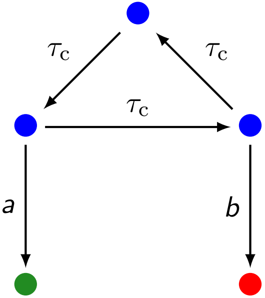

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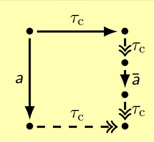

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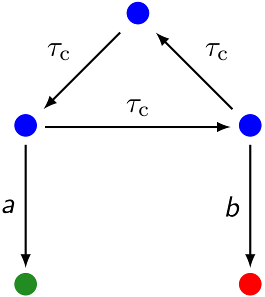

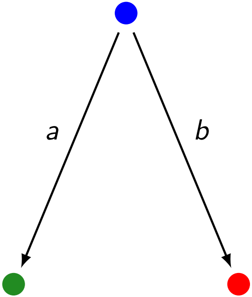

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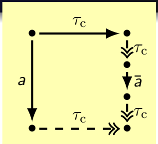

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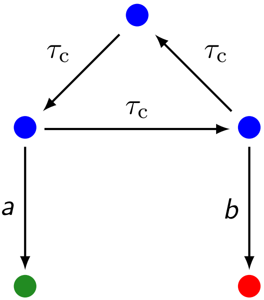
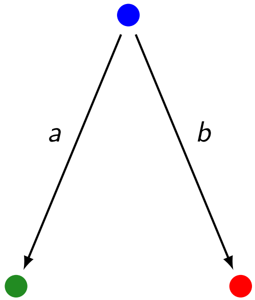
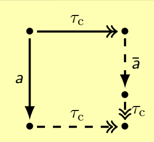


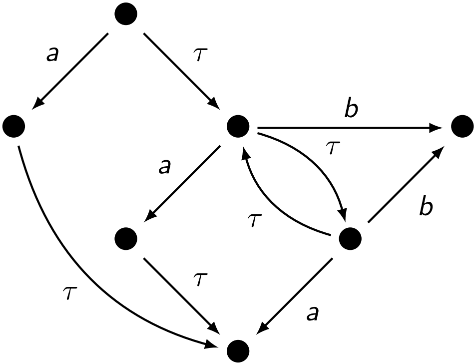


Here we used the equivalence classes of $\mathcal{A}/\overset{\tau_c}{\twoheadleftarrow\!\!\twoheadrightarrow}$ as nodes.
(None of the blue nodes could be chosen as representative, as none of them can done both an a an a b transition.)

Examples



Reduction based on confluence
using representatives



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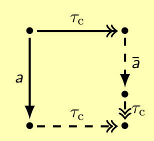

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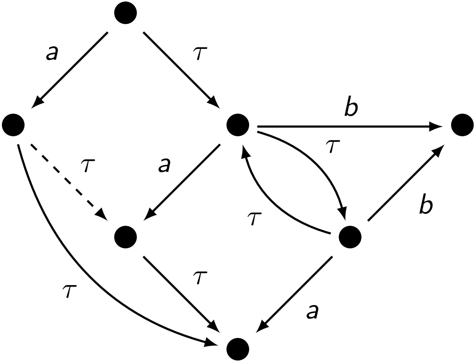

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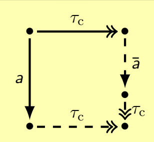

Reduction based on confluence
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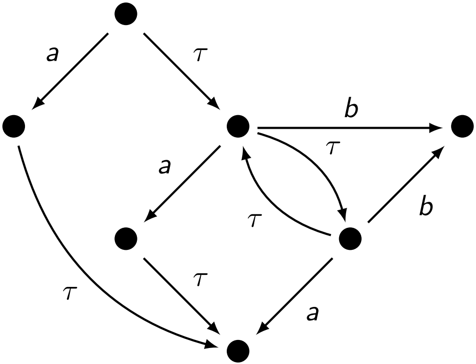

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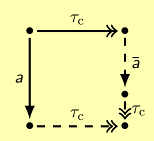

Reduction based on confluence
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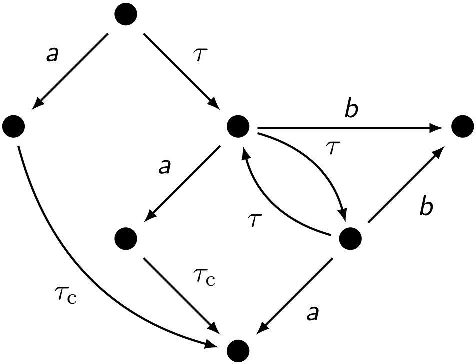

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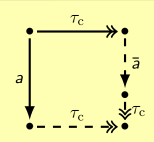

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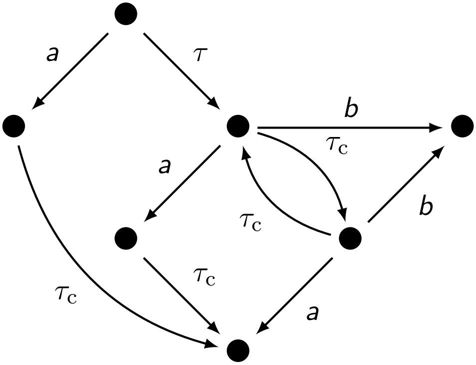

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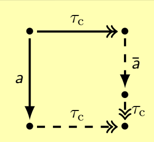

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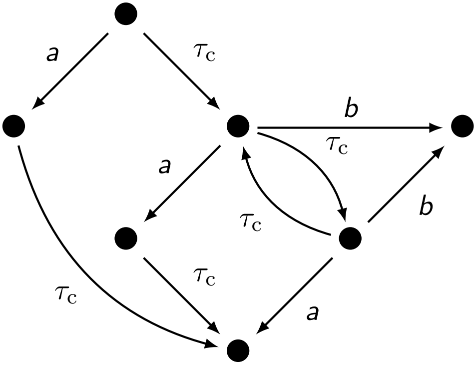

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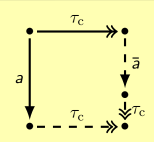

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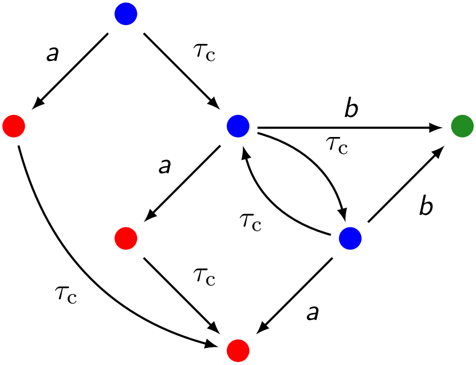

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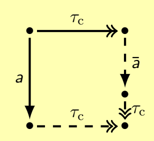

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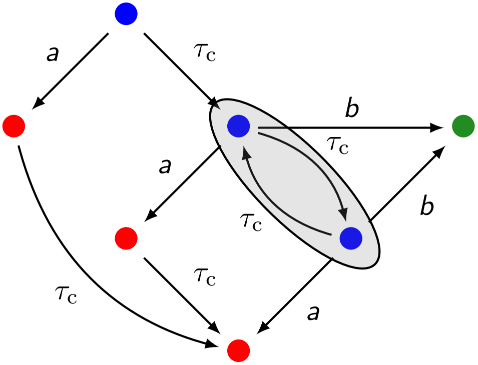

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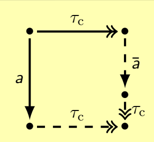

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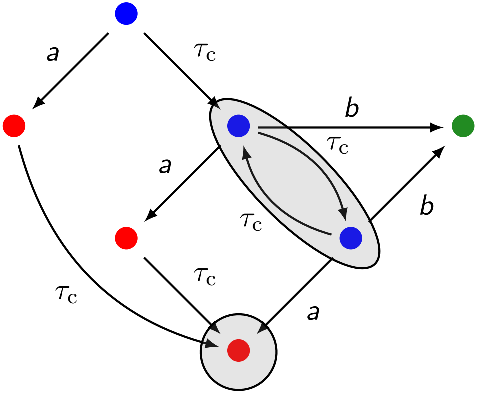
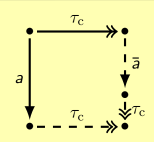


Examples



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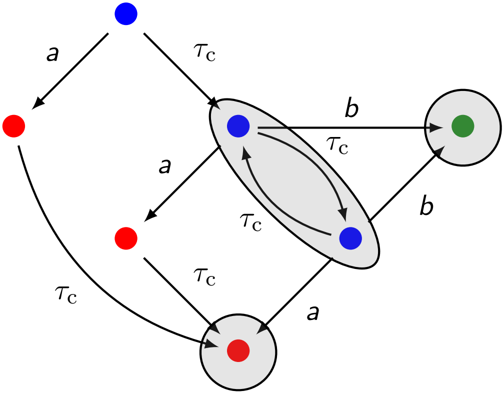

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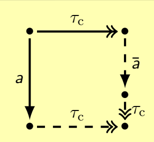

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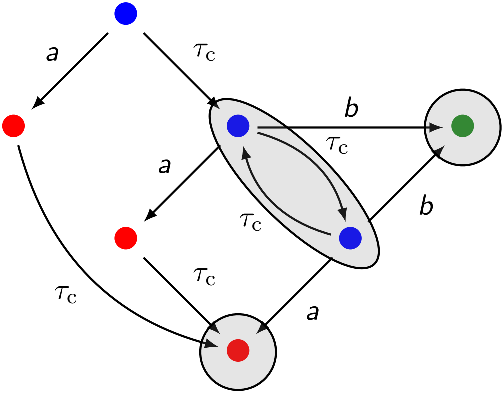
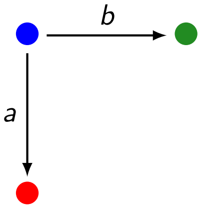
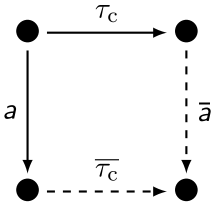




Confluence for probabilistic automata

For simplicity we only consider strong confluence from now on.

Non-probabilistic:



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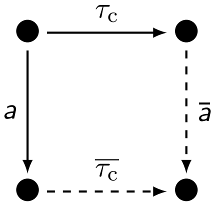
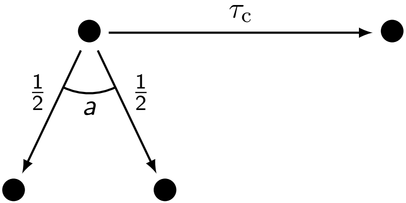


Probabilistic:



Confluence for probabilistic automata

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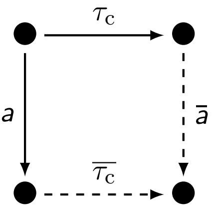
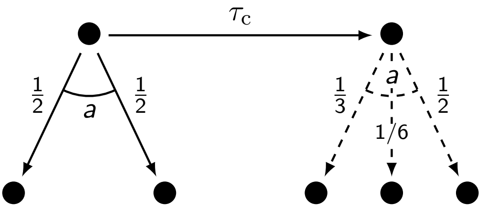
Non-probabilistic:



Probabilistic:



Confluence for probabilistic automata

For simplicity we only consider strong confluence from now on.

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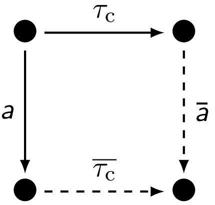
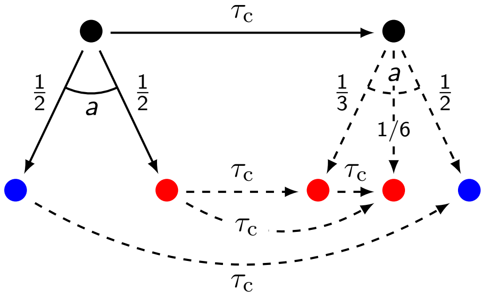
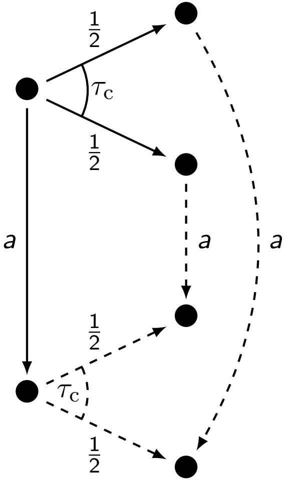


Probabilistic:



Why τ_c steps should have a Dirac distribution



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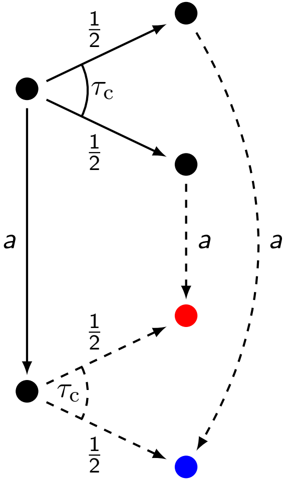

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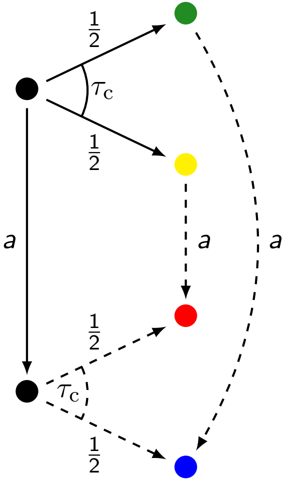

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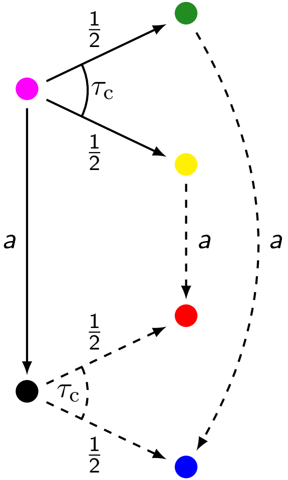

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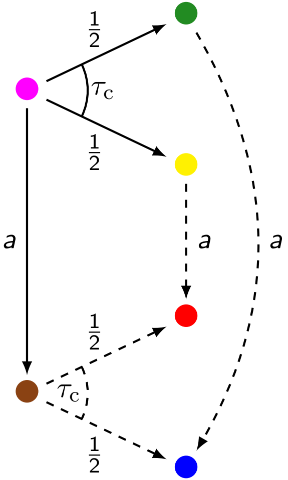

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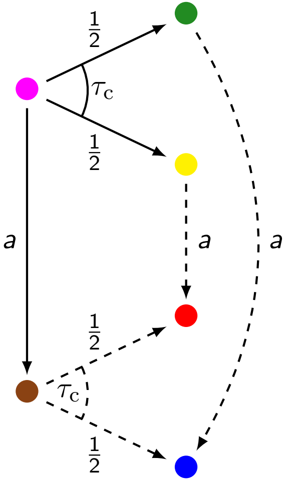

As all states are (potentially) different, no reduction can be obtained.

Detecting confluence using LPPEs

Given an LPPE, confluence can be detected using theorem proving.

$$X(\vec{g} : \vec{G}) = \sum_{\vec{d_1}:\vec{D_1}} c_1 \Rightarrow \tau \sum\!\!\!\!\bullet_{\vec{e_1}:\vec{E_1}} f_1 : X(\vec{n_1})$$
$$\dots$$
$$+ \sum_{\vec{d_k}:\vec{D_k}} c_k \Rightarrow a_k(b_k) \sum\!\!\!\!\bullet_{\vec{e_k}:\vec{E_k}} f_k : X(\vec{n_k})$$

To check the first τ-summand is confluent, we check whether indeed

- $|E_1| = 1$, or $f_1 = 1.0$ for one $e_i \in E_1$.
- the summand is confluent with all other summands.

Detecting confluence using LPPEs

$$
\begin{aligned}
X(\vec{g} : \vec{G}) = & \sum_{\vec{d_1}:\vec{D_1}} c_1 \Rightarrow \tau \cdot X(\vec{n_1}) \\
& \cdots \\
& + \sum_{\vec{d_k}:\vec{D_k}} c_k \Rightarrow a_k(b_k) \sum\!\!\!\!\bullet_{\vec{e_k}:\vec{E_k}} f_k : X(\vec{n_k})
\end{aligned}
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To prove:

$$c_1(g, d_1) \wedge c_k(g, d_k) \quad \rightarrow$$

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$$
\left(\quad c_k(n_1(g, d_1), d_k) \right.
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$$
\left(\begin{array}{ll} & c_k(n_1(g, d_1), d_k) \\ \wedge & c_1(n_k(g, d_k, e_k), d_1) \\ & \end{array} \right.
$$

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$$

$$
\left(\begin{array}{ll}
 & c_k(n_1(g, d_1), d_k) \\
\wedge & c_1(n_k(g, d_k, e_k), d_1) \\
\wedge & a_k(g, d_k) = a_k(n_1(g, d_1), d_k)
\end{array} \right.
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\end{array} \right)
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Reducing LPPEs based on confluent τ steps

After τ_c steps have been identified, two types of reductions are possible:

1. Symbolic prioritisation: change the LPPE
   - Let c be a confluent summand
   - Find a non-confluent summand n such that c is always enabled after executing n
   - Change the next state of n, basically merging n and c

As we only do this for non-confluent summands, loops are avoided.

Reducing LPPEs based on confluent τ steps

After τ_c steps have been identified, two types of reductions are possible:

1. Symbolic prioritisation: change the LPPE
   - Let *c* be a confluent summand
   - Find a non-confluent summand *n* such that *c* is always enabled after executing *n*
   - Change the next state of *n*, basically merging *n* and *c*

   As we only do this for non-confluent summands, loops are avoided.
2. On-the-fly state space generation using representatives
   - Generate the state space from the LPPE
   - For each transition that is visited, go to the representative of the target state
   - When no representative it known yet, compute it (using a variation on Tarjan's SCC algoritm)

Contents

Conclusions and Future Work

Conclusions / Results

- We developed the process algebra prCRL, incorporating both data and probability.
- We defined a normal form for prCRL, the LPPE; starting point for symbolic optimisations and easy state space generation.
- We generalised reduction techniques from LPEs to the probabilistic case; constant elimination, confluence reduction

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Future work

- Finish work on confluence reduction: proofs, case study, implementation
- Develop additional reduction techniques
- Generalise proof techniques such as cones and foci to the probabilistic case

Questions

Questions?