UNIVERSITY OF TWENTE.

Formal Methods & Tools.

SCOOP: A Tool for SymboliC Optimisations Of Probabilistic Processes

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The context – probabilistic model checking

Probabilistic model checking:

- Verifying quantitative properties,
- Using a probabilistic model (e.g., a probabilistic automaton)

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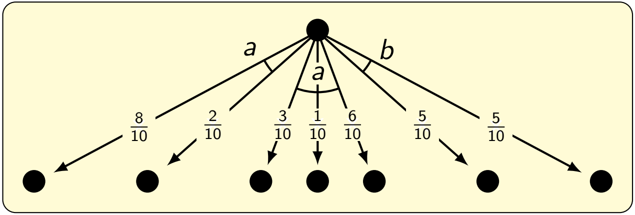

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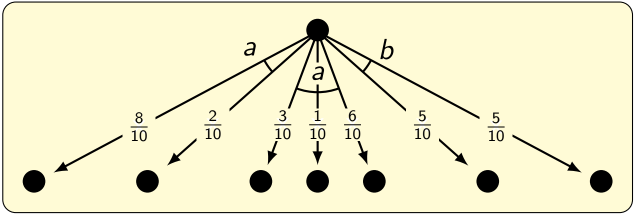

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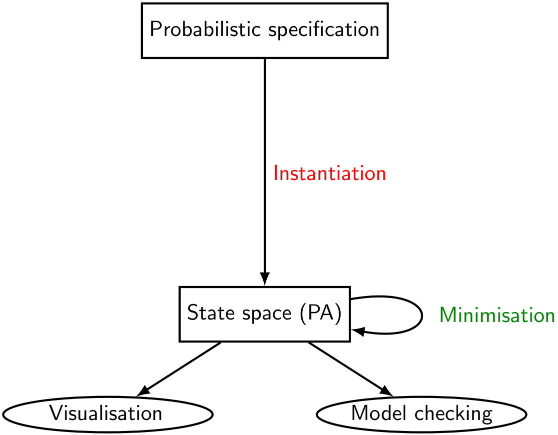
Limitations of previous approaches:

- Restricted treatment of data
- Susceptible to the state space explosion problem

Overview of our approach



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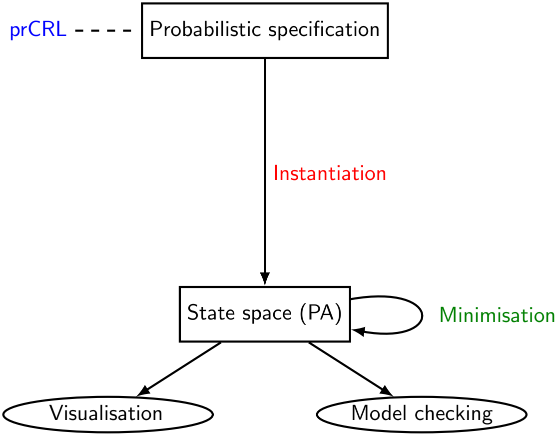

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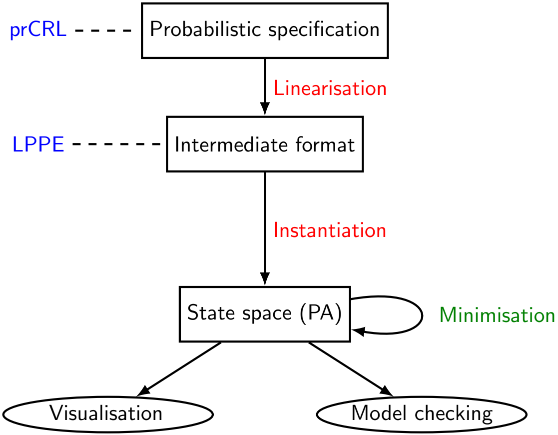

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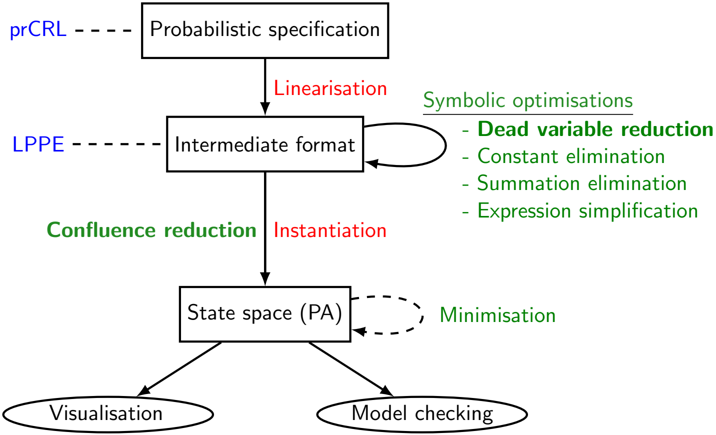

The input language: prCRL

Specification language prCRL:

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- Semantics defined in terms of probabilistic automata

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Basic grammar

$$p ::= Y(t) \mid c \Rightarrow p \mid p + p \mid \sum_{x:D} p \mid a(t) \sum\!\!\!\bullet_{x:D} f : p$$

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For composability we introduced extended prCRL. It extends prCRL by parallel composition, encapsulation, hiding and renaming.

A linear format for prCRL: the LPPE

LPPEs are a subset of prCRL specifications:

$$
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X(\boldsymbol{g} : \boldsymbol{G}) = & \sum_{\boldsymbol{d_1}:\boldsymbol{D_1}} c_1 \Rightarrow a_1 \sum\!\!\!\!\bullet_{\boldsymbol{e_1}:\boldsymbol{E_1}} f_1 : X(\boldsymbol{n_1}) \\
& \dots \\
& + \sum_{\boldsymbol{d_k}:\boldsymbol{D_k}} c_k \Rightarrow a_k \sum\!\!\!\!\bullet_{\boldsymbol{e_k}:\boldsymbol{E_k}} f_k : X(\boldsymbol{n_k})
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Advantages of using LPPEs instead of prCRL specifications:

- Easy state space generation
- Straight-forward parallel composition
- **Symbolic optimisations enabled at the language level**

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Theorem

Every specification (without unguarded recursion) can be linearised to an LPPE, preserving strong probabilistic bisimulation.

Optimisation techniques

1. LPPE simplification techniques
    - Constant elimination
    - Summation elimination
    - Expression simplification

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$X(id : Id) = print(id) \cdot X(id)$

init $X(Mark)$

$\rightarrow$

$X = print(Mark) \cdot X$

init X

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$X = \sum_{d:\{1,2,3\}} d = 2 \Rightarrow send(d) \cdot X$

init X

$\rightarrow$

$X = send(2) \cdot X$

init X

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$X = (3 = 1 + 2 \vee x > 5) \Rightarrow \mathit{beep} \cdot Y$ $\rightarrow$ $X = \mathit{beep} \cdot Y$

Optimisation techniques

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- Deduce the control flow of an LPPE
- Examine relevance (liveness) of variables
- Reset dead variables

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- Detect confluent internal transitions
- Give these transitions priority

SCOOP

$$\text{prCRL} \xrightarrow{\textit{linearise}} \text{LPPE} \xrightarrow{\textit{optimise}} \text{LPPE} \xrightarrow{\textit{instantiate}} \text{AUT/PRISM}$$

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- Open source, publicly available (6640 lines of Haskell code)
- Stand-alone downloadable tool and web-based interface

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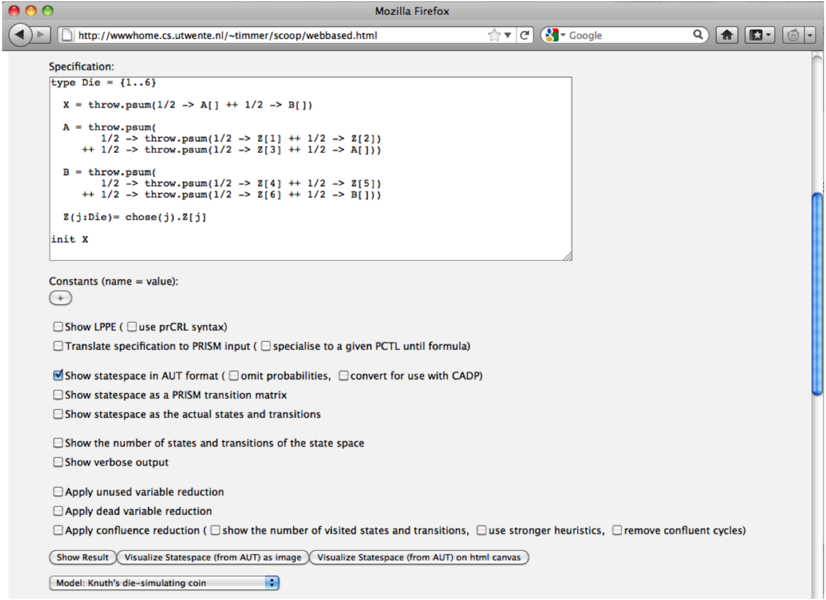
Spec.	Original States	Reduced States	Visited States	Running time (sec) Before	After
leader-3-15	1,043,635	68,926	251,226	313.35	65.96
leader-3-18	2,028,181	118,675	428,940	1161.58	124.74
leader-3-21	out of mem.	187,972	675,225	–	205.90
leader-3-27	out of mem.	398,170	1,418,220	–	497.94
leader-4-5	759,952	61,920	300,569	322.62	75.14
leader-4-6	1,648,975	127,579	608,799	1073.16	155.74
leader-4-7	out of mem.	235,310	1,108,391	–	291.25
leader-4-8	out of mem,	400,125	1,865,627	–	1069.56
leader-5-2	260,994	14,978	97,006	155.37	29.40
leader-5-3	out of mem.	112,559	694,182	–	213.10

Screenshot


Mozilla Firefox
http://wwwhome.cs.utwente.nl/~timmer/scoop/webbased.html
Google
Specification:
type Die = {1..6}
X = throw.psum(1/2 -> A[] ++ 1/2 -> B[])
A = throw.psum(
1/2 -> throw.psum(1/2 -> Z[1] ++ 1/2 -> Z[2])
++ 1/2 -> throw.psum(1/2 -> Z[3] ++ 1/2 -> A[]))
B = throw.psum(
1/2 -> throw.psum(1/2 -> Z[4] ++ 1/2 -> Z[5])
++ 1/2 -> throw.psum(1/2 -> Z[6] ++ 1/2 -> B[]))
Z(j:Die)= chose(j).Z[j]
init X
Constants (name = value):
+
Show LPPE (use prCRL syntax)
Translate specification to PRISM input (specialise to a given PCTL until formula)
Show statespace in AUT format (omit probabilities, convert for use with CADP)
Show statespace as a PRISM transition matrix
Show statespace as the actual states and transitions
Show the number of states and transitions of the state space
Show verbose output
Apply unused variable reduction
Apply dead variable reduction
Apply confluence reduction (show the number of visited states and transitions, use stronger heuristics, remove confluent cycles)
Show Result
Visualize Statespace (from AUT) as image
Visualize Statespace (from AUT) on html canvas
Model: Knuth's die-simulating coin

Questions

Questions?

For more information, and the tool itself, go to

`http://wwwhome.cs.utwente.nl/~timmer/scoop/`