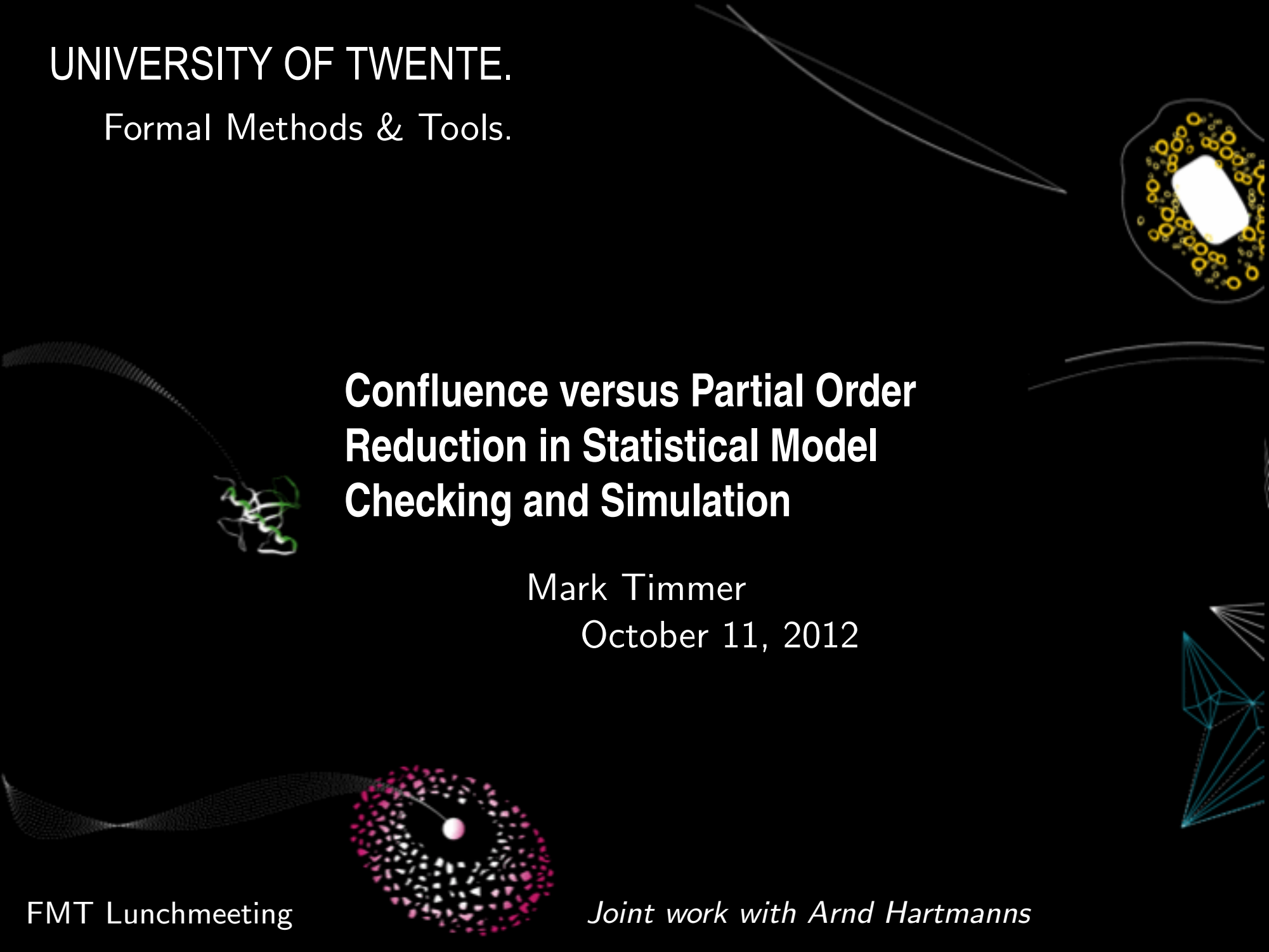
UNIVERSITY OF TWENTE.

Formal Methods & Tools.



Confluence versus Partial Order Reduction in Statistical Model Checking and Simulation

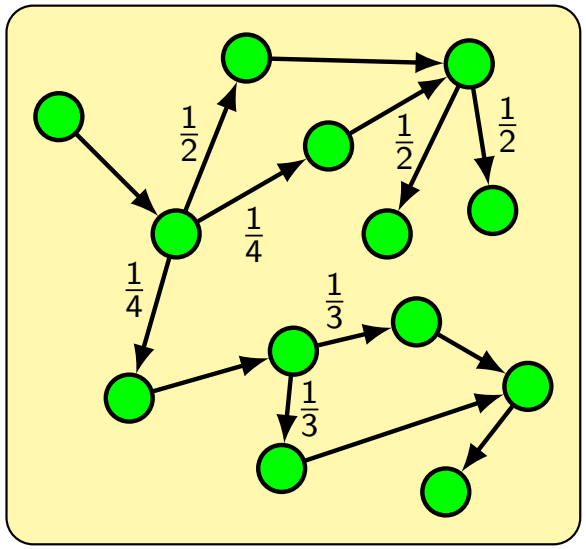
Mark Timmer

October 11, 2012

FMT Lunchmeeting

Joint work with Arnd Hartmanns

(Statistical) Model Checking



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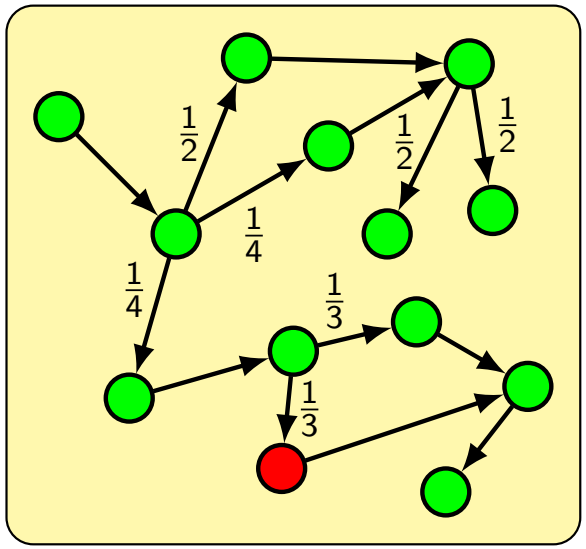

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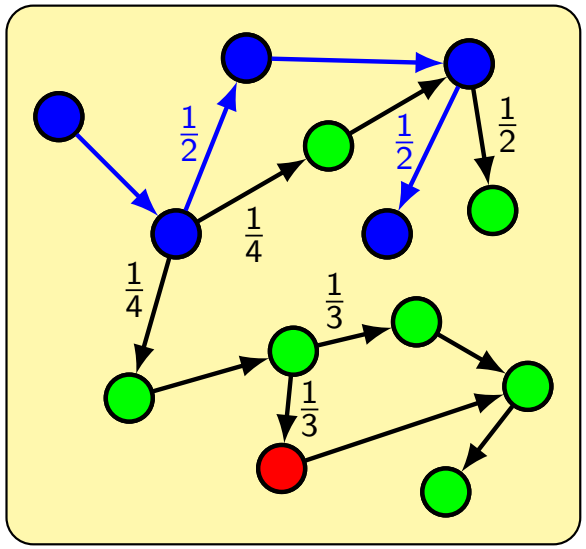

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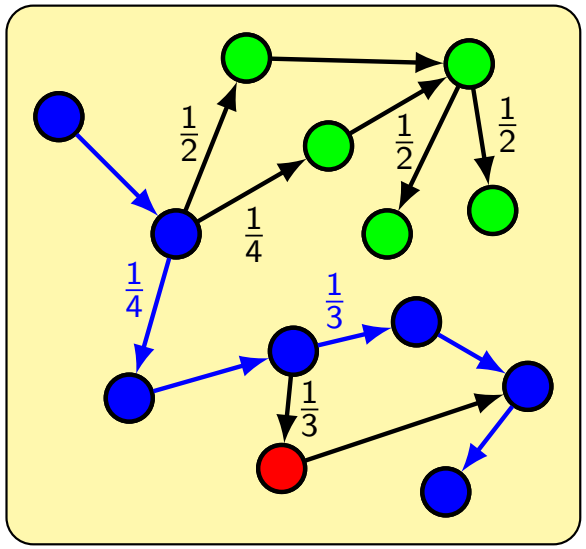

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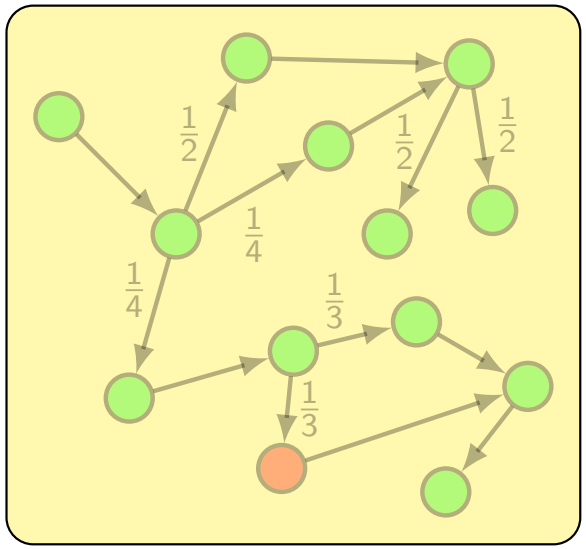

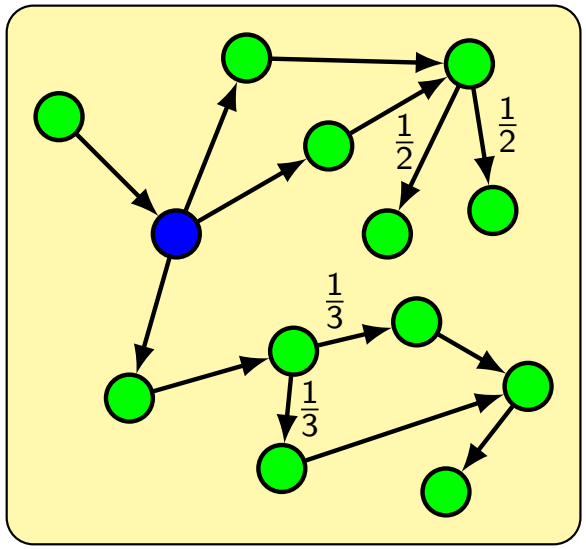

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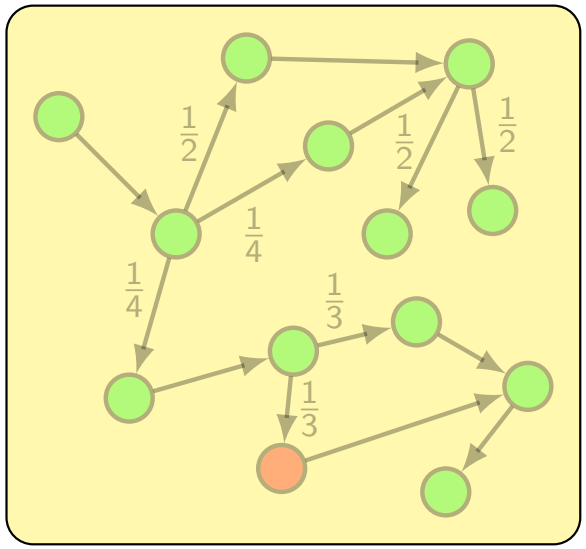

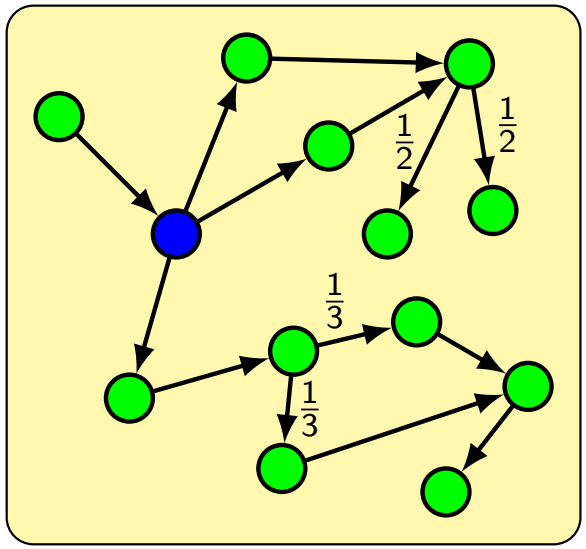

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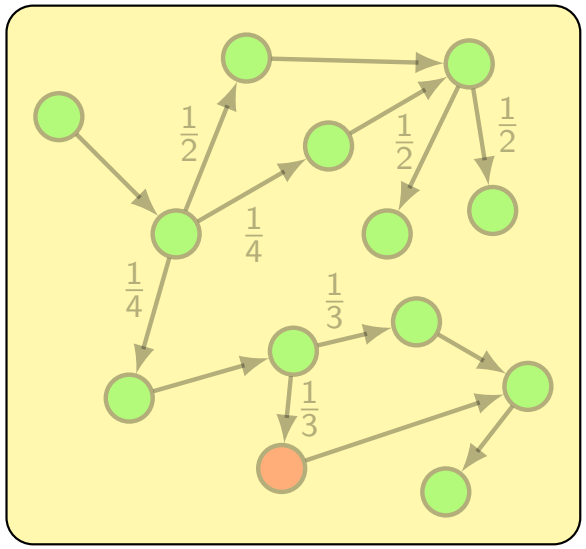

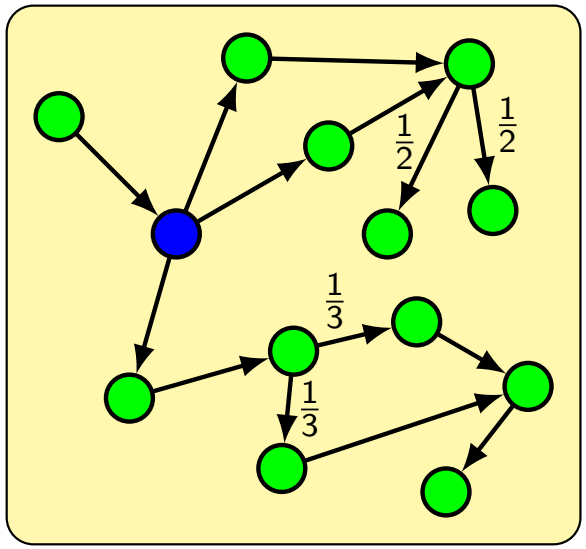

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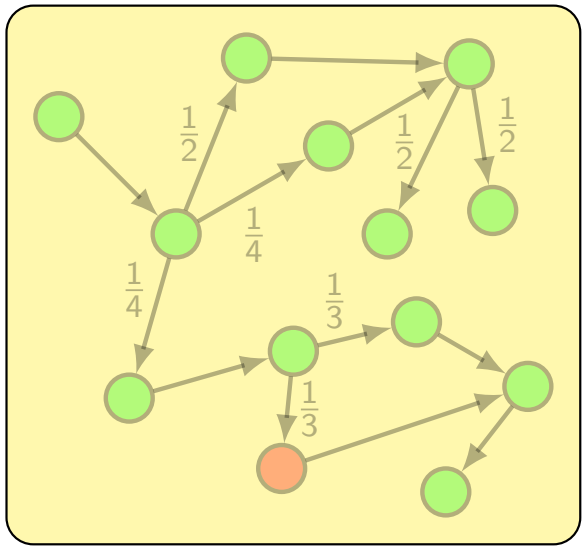

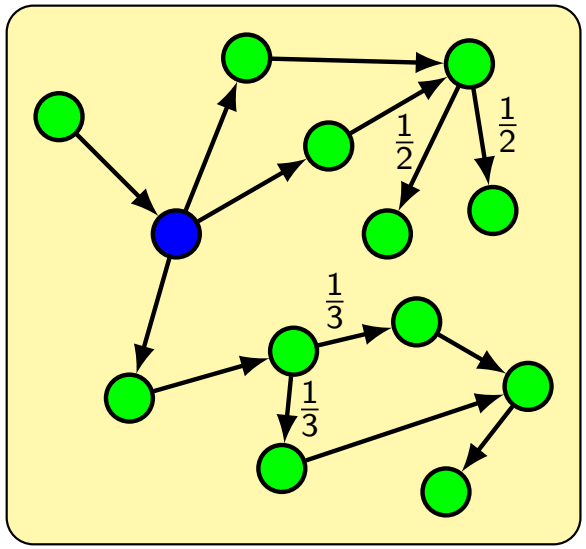

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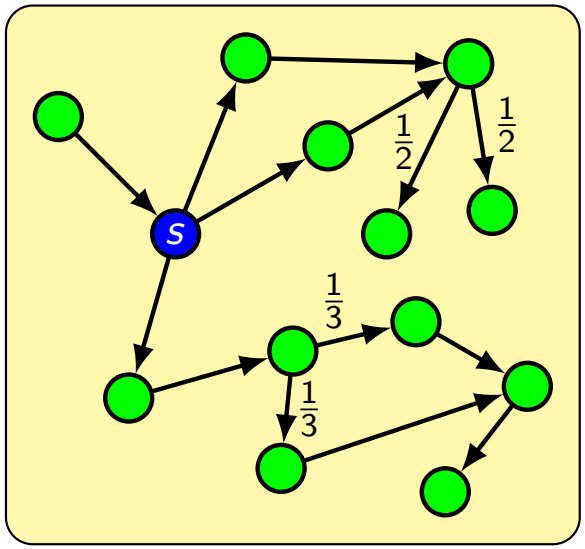

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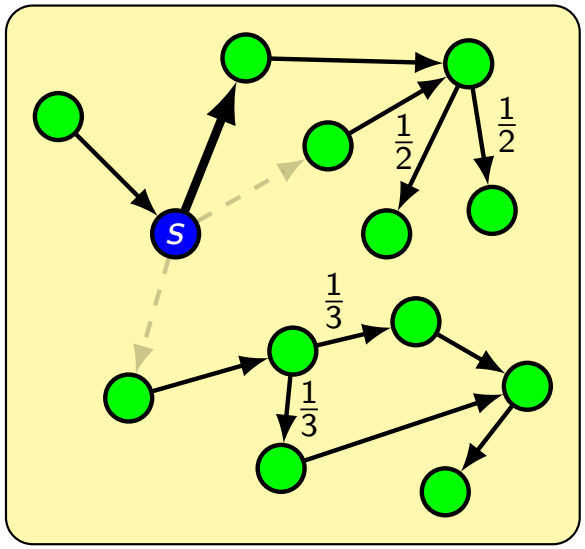

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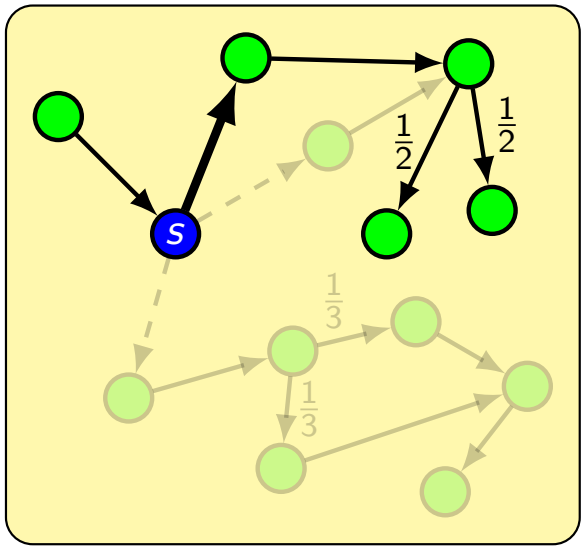

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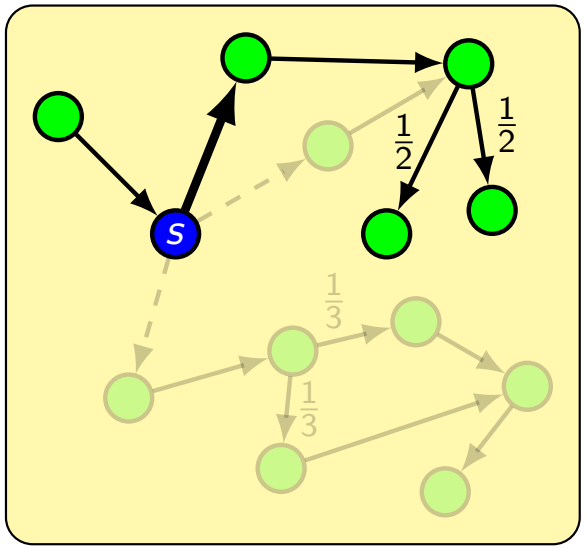

Important:

reduced system should be equivalent

The model: Probabilistic Automata

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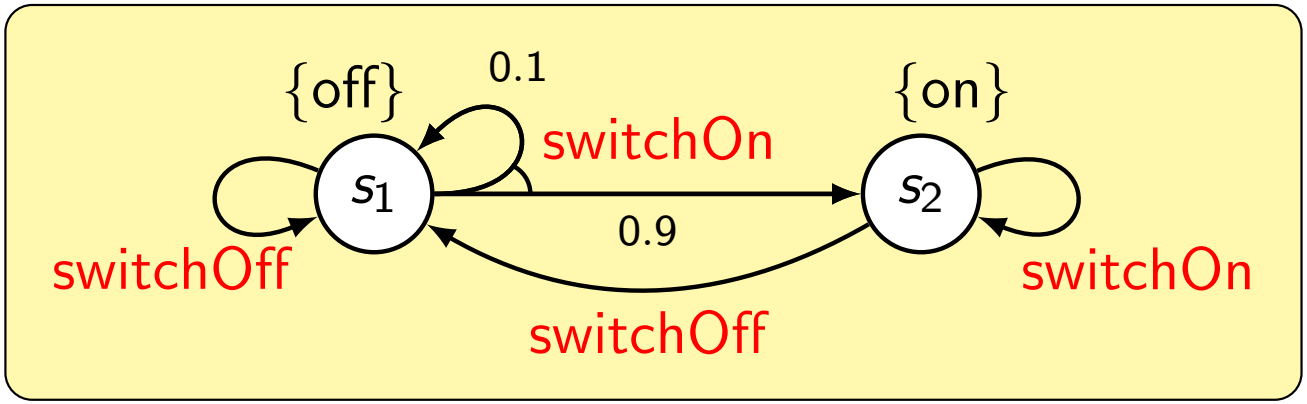

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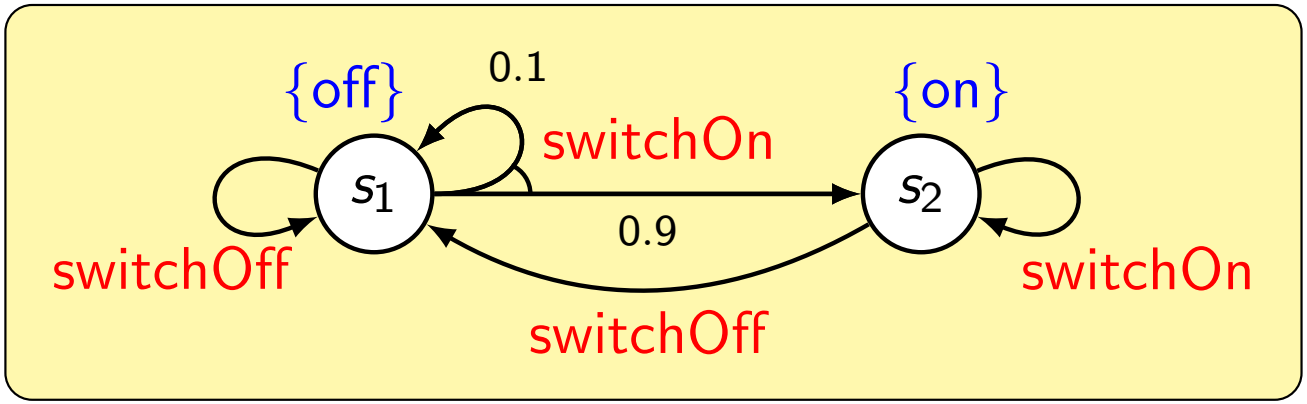

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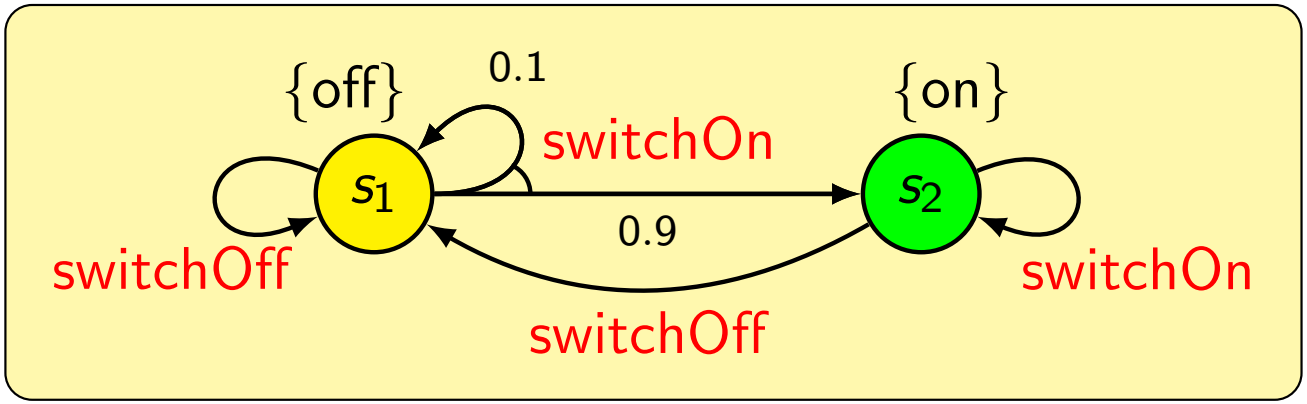

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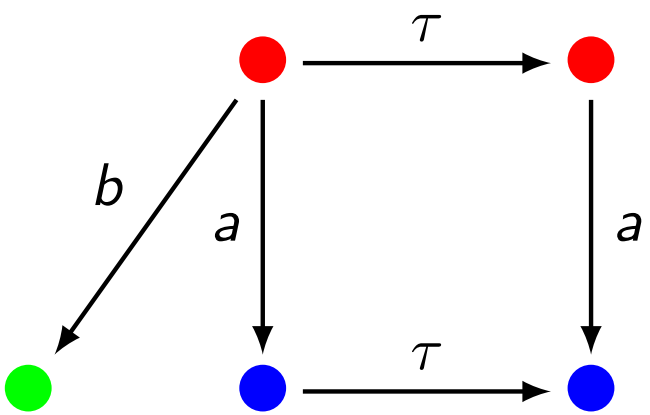

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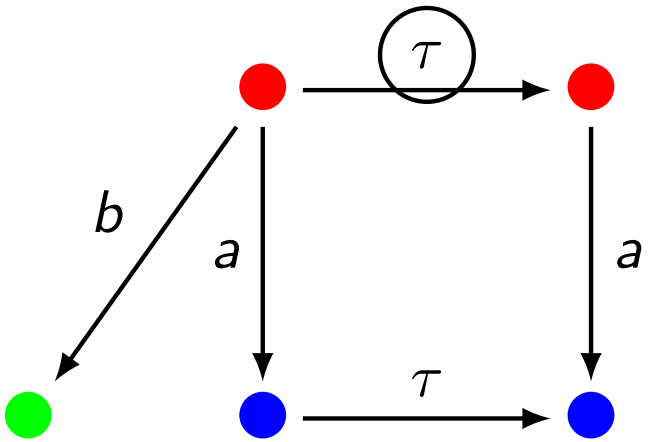

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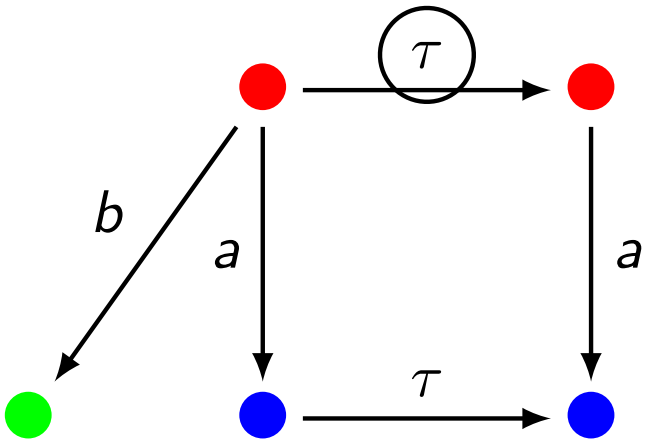

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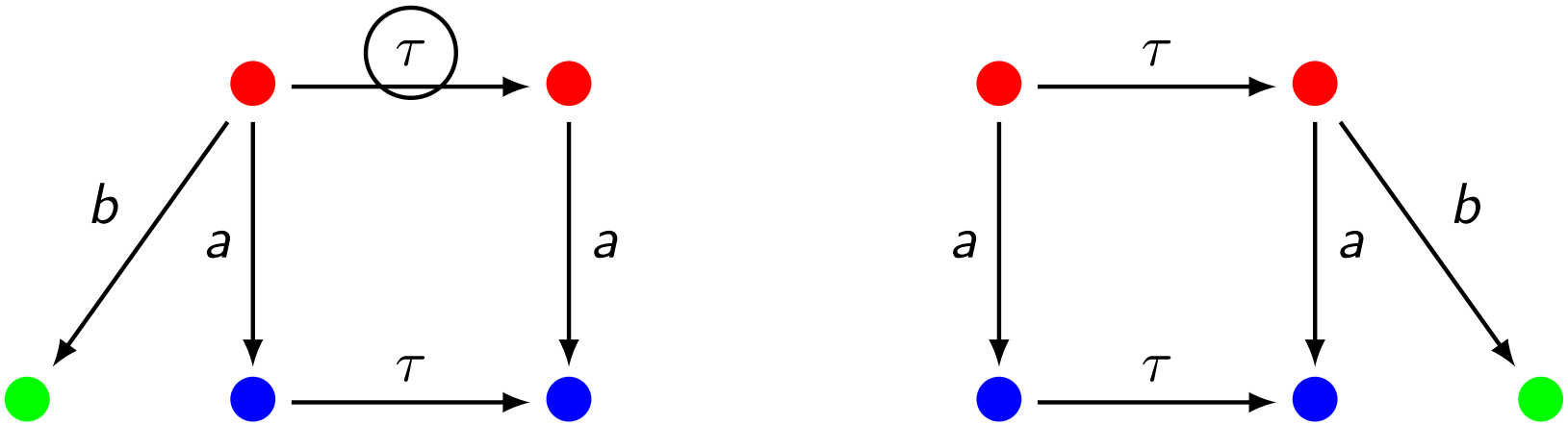

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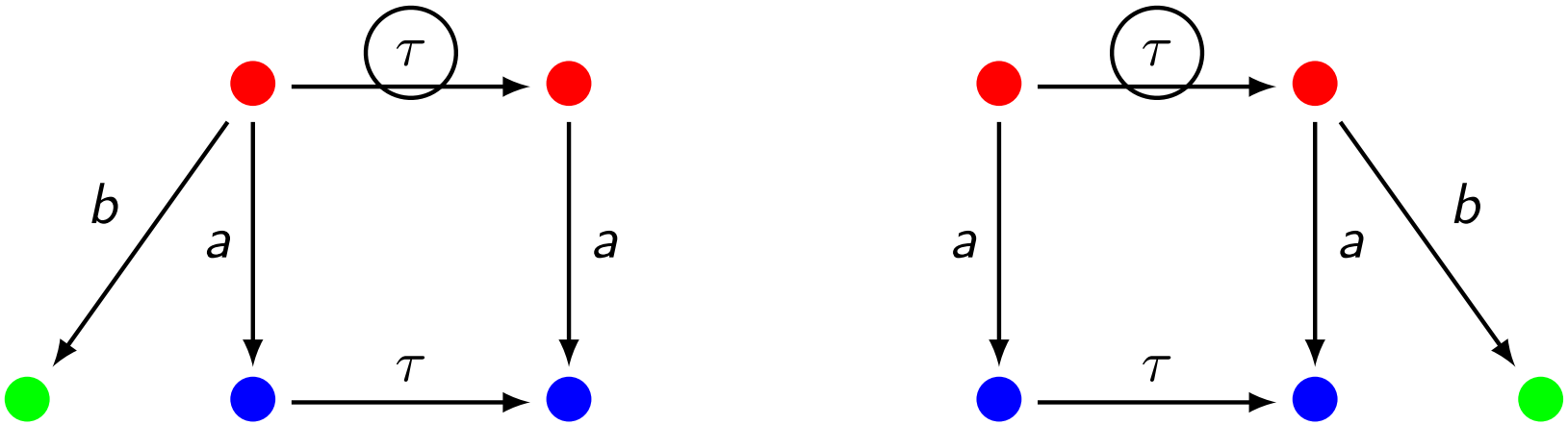

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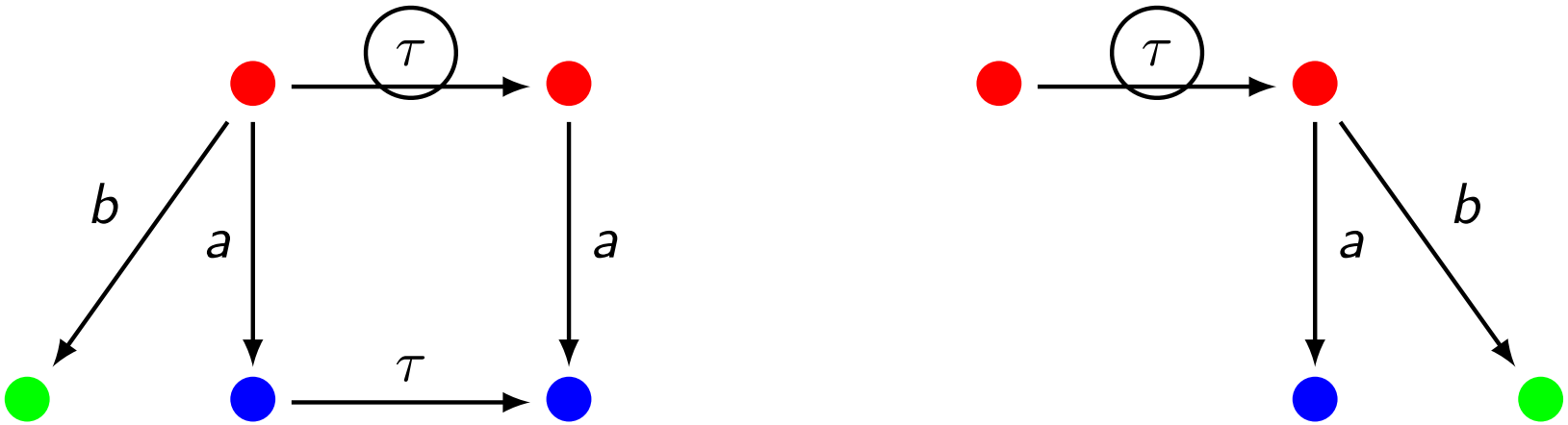

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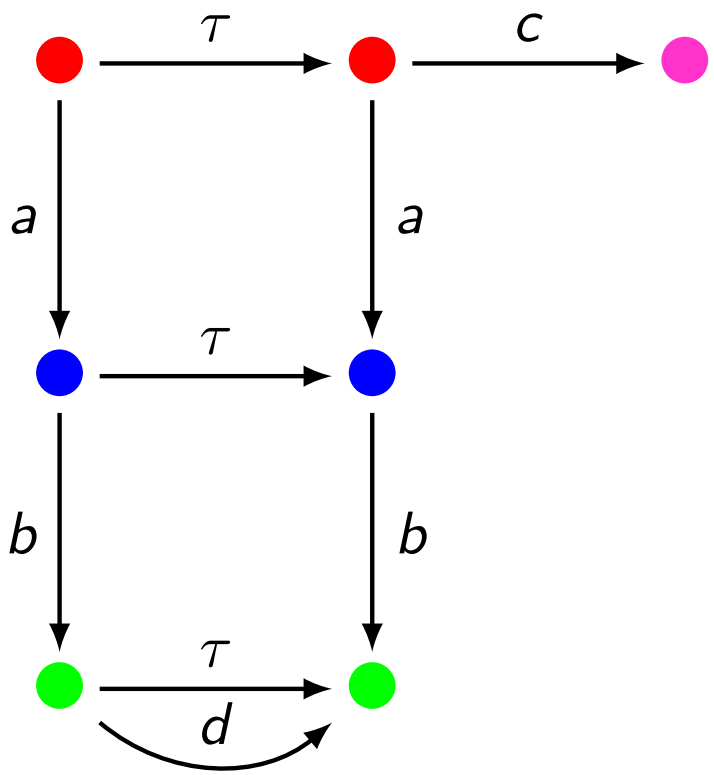

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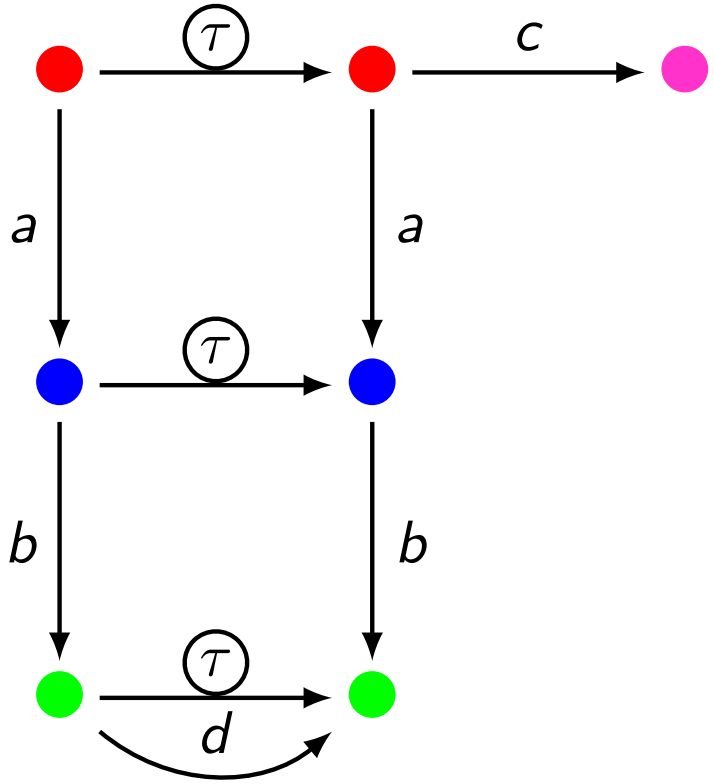

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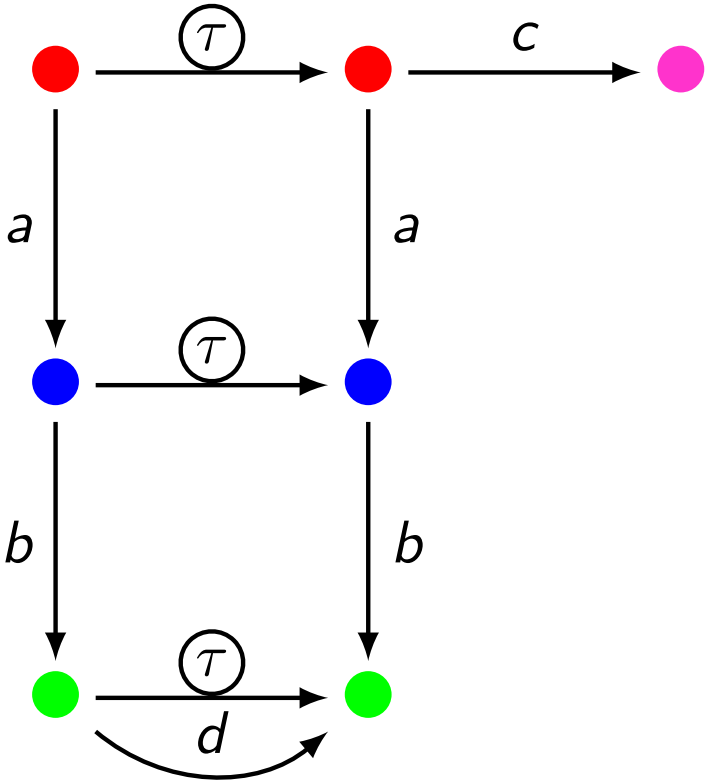

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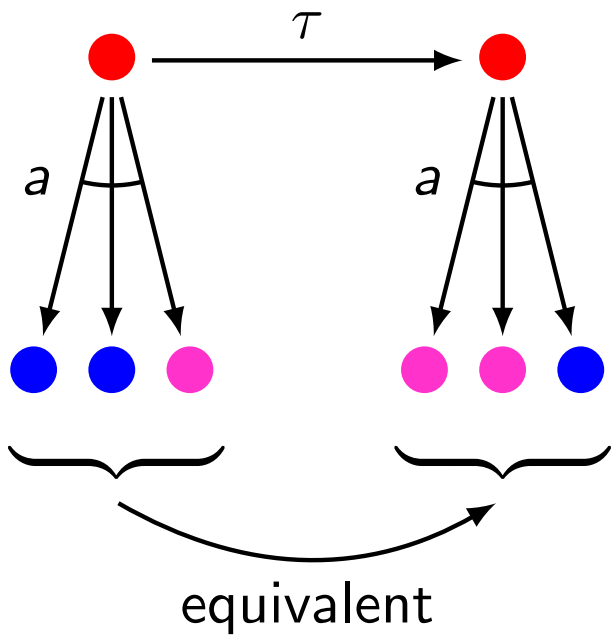

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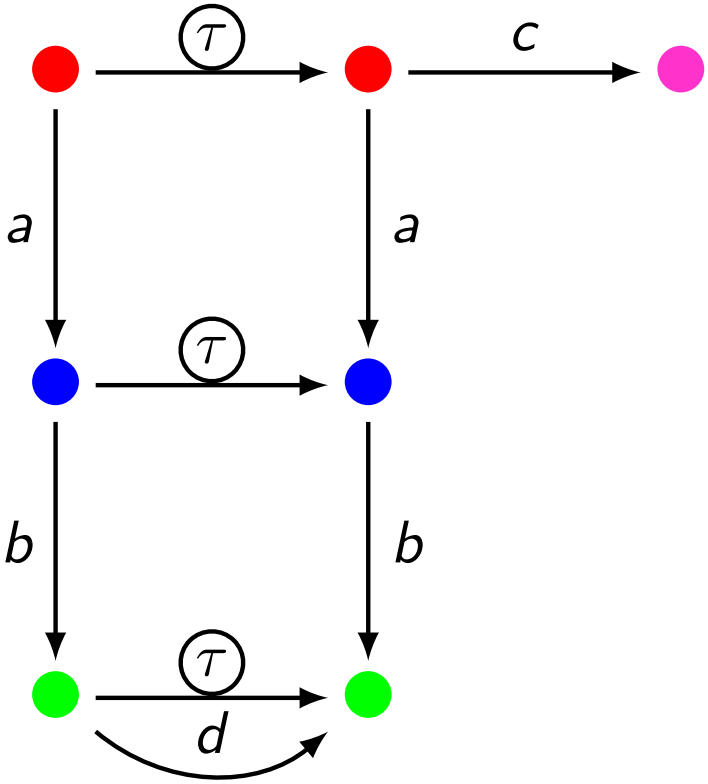

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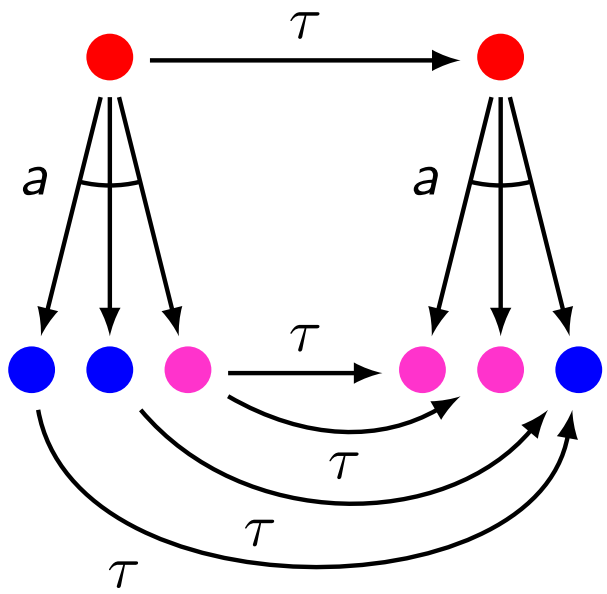

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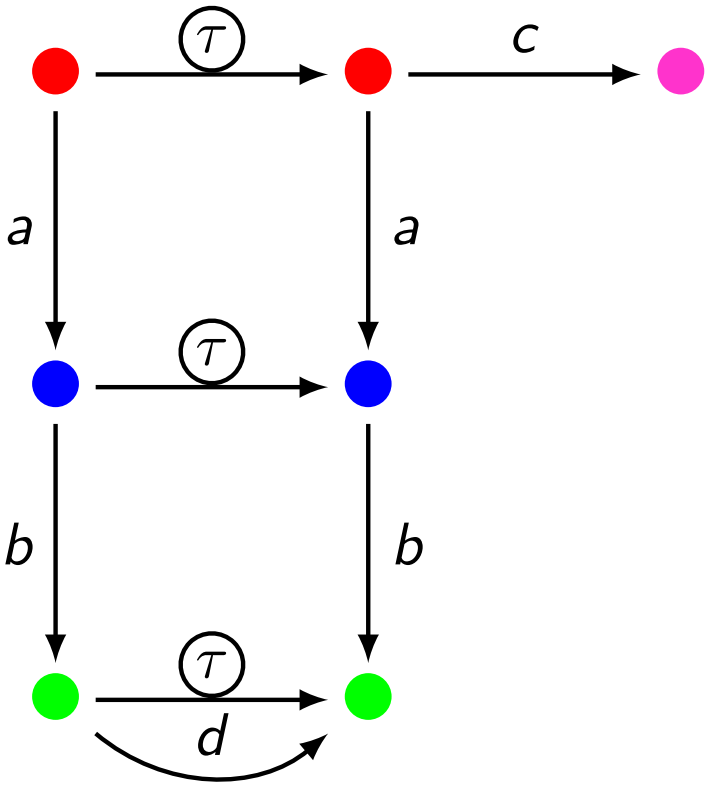

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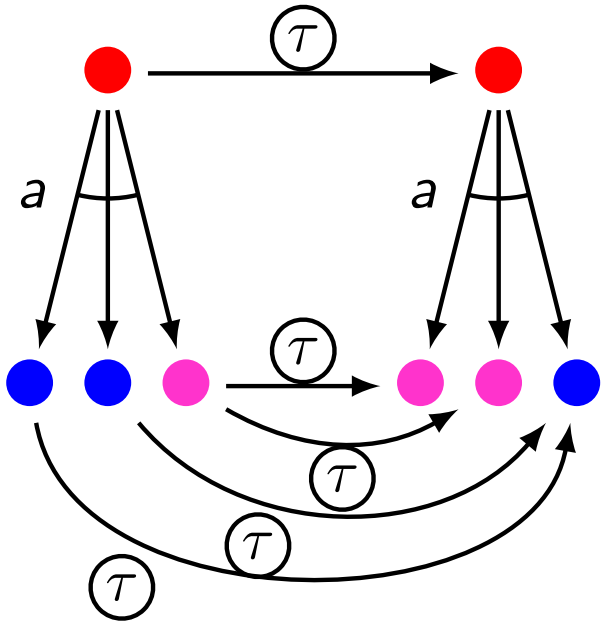

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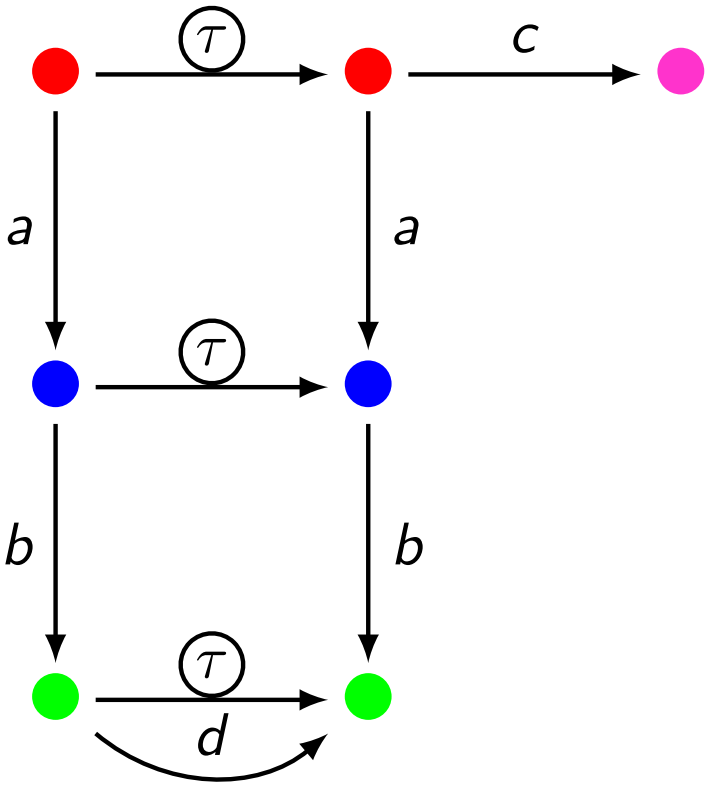

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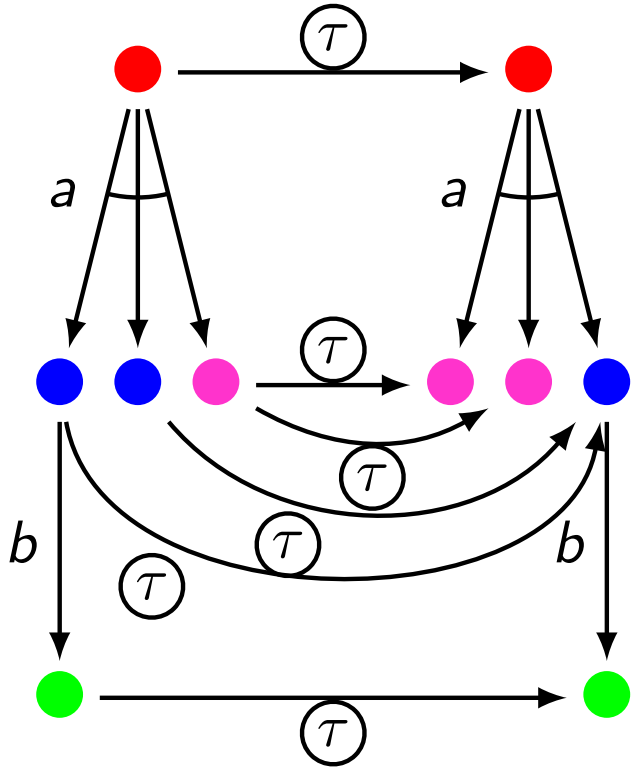

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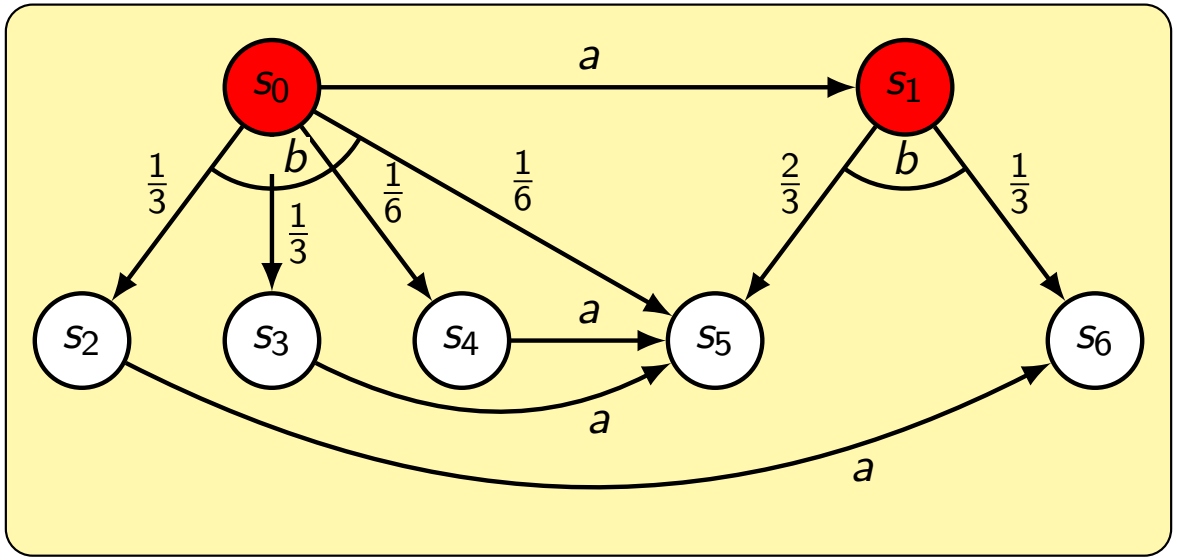

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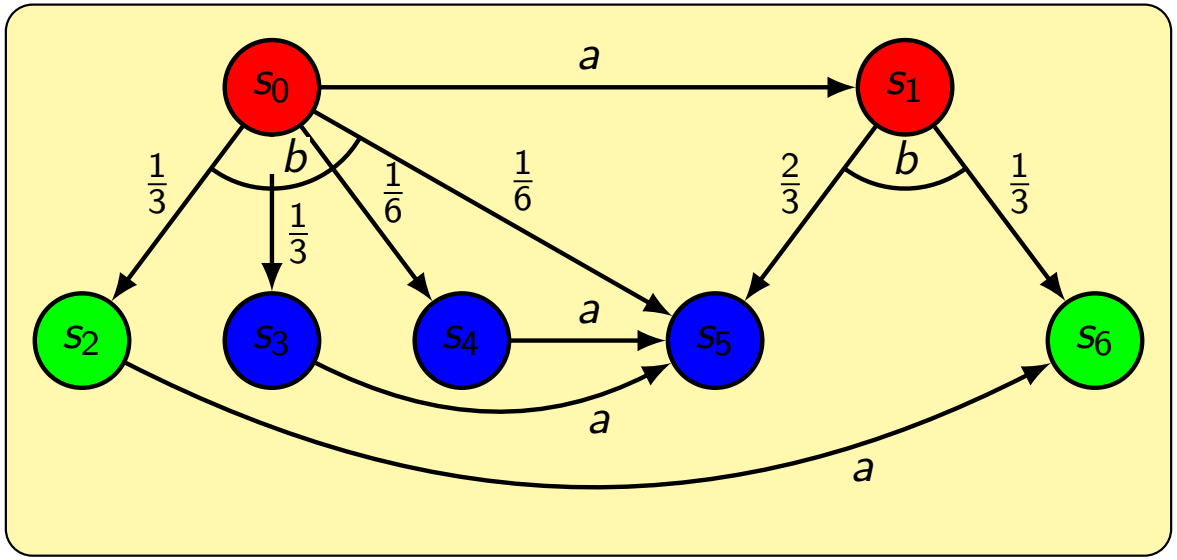

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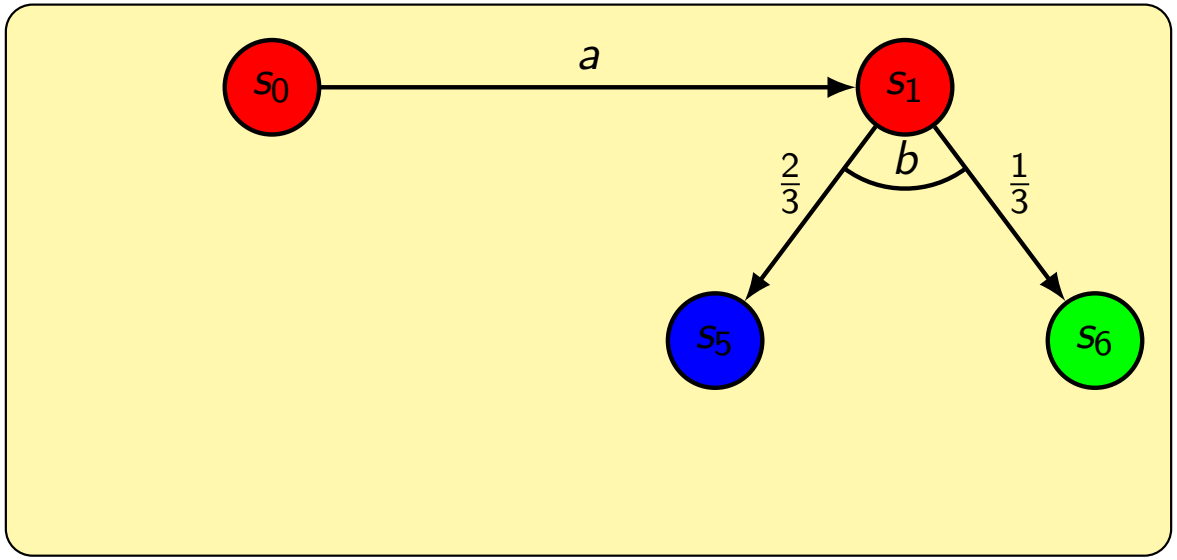

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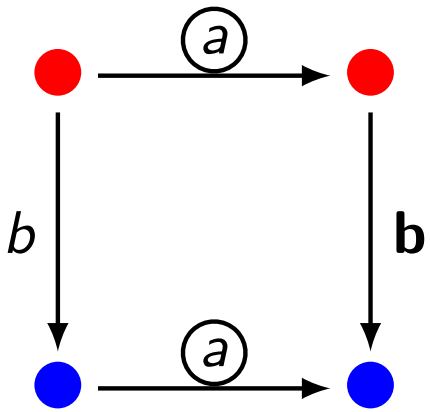

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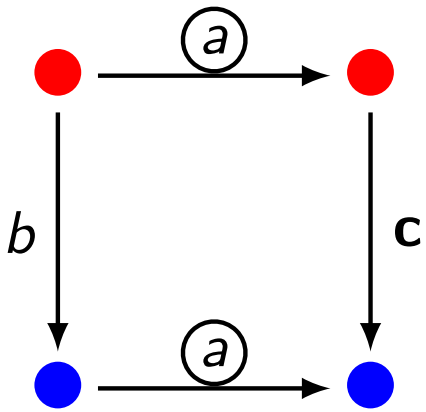

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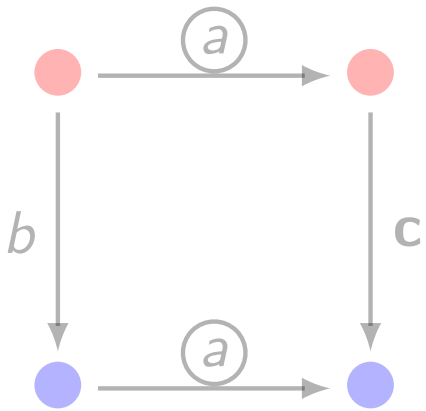

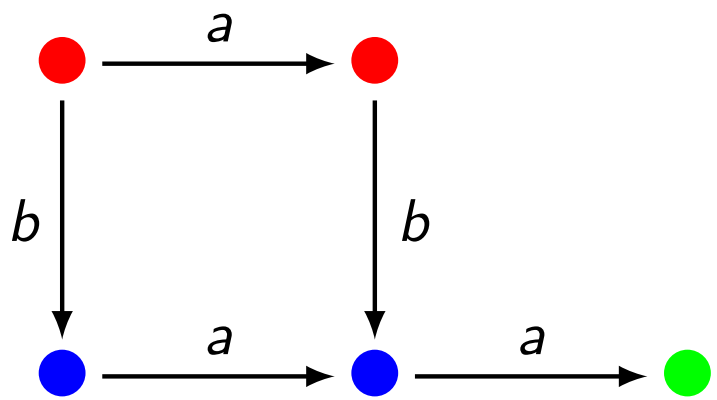

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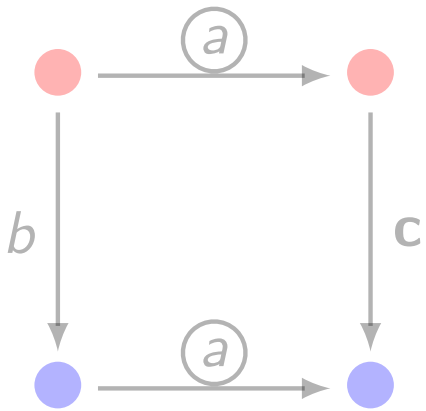

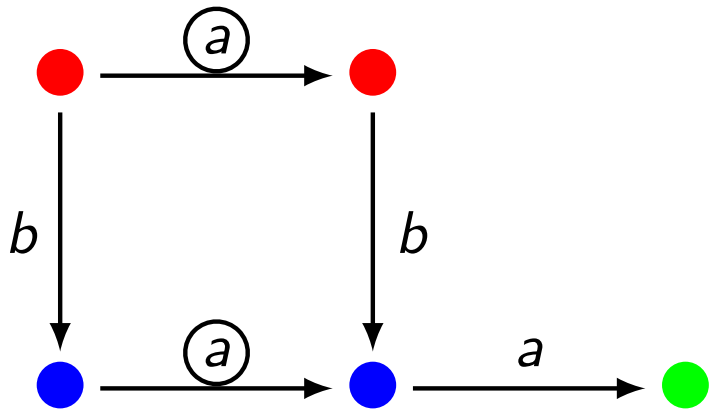

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Distributions μ and ν are $\mathcal{T}$-equivalent, if $\mu \equiv_R \nu$ for the smallest equivalence relation R containing the set

$$\{(s,t) \mid s \in \mathrm{spt}(\mu), t \in \mathrm{spt}(\nu), s \xrightarrow{a} t \in \mathcal{T}\}$$

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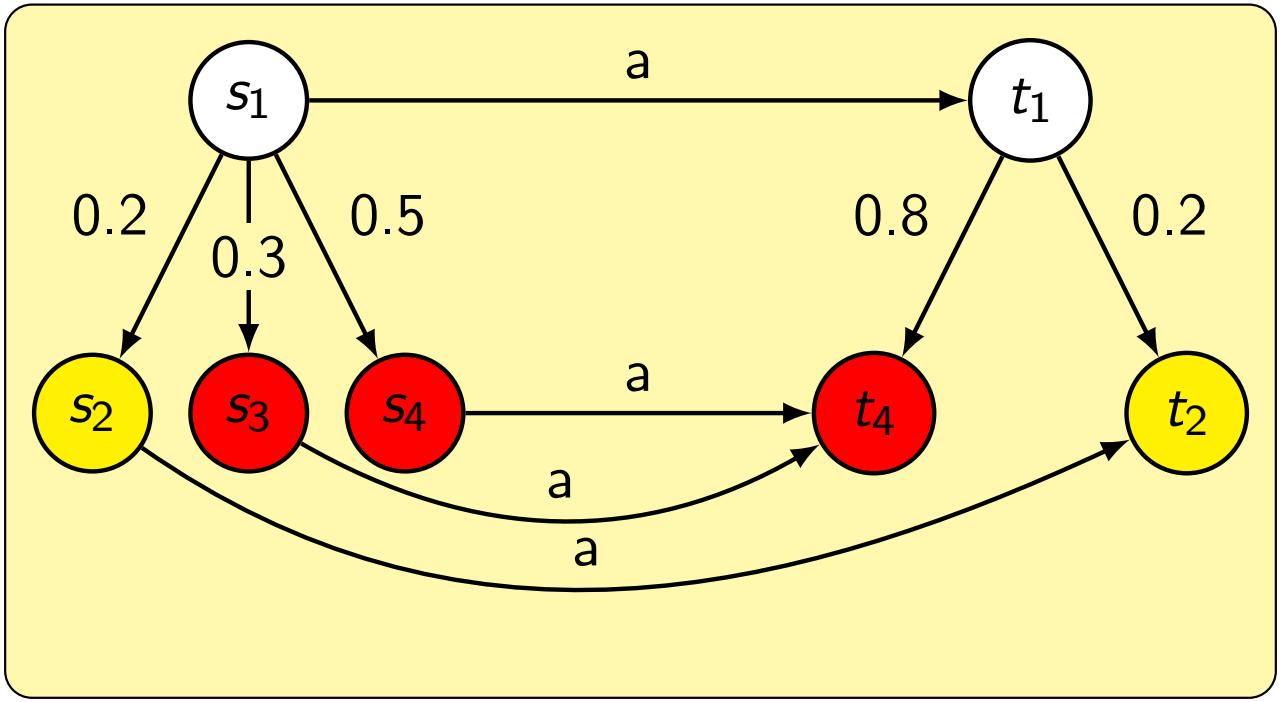

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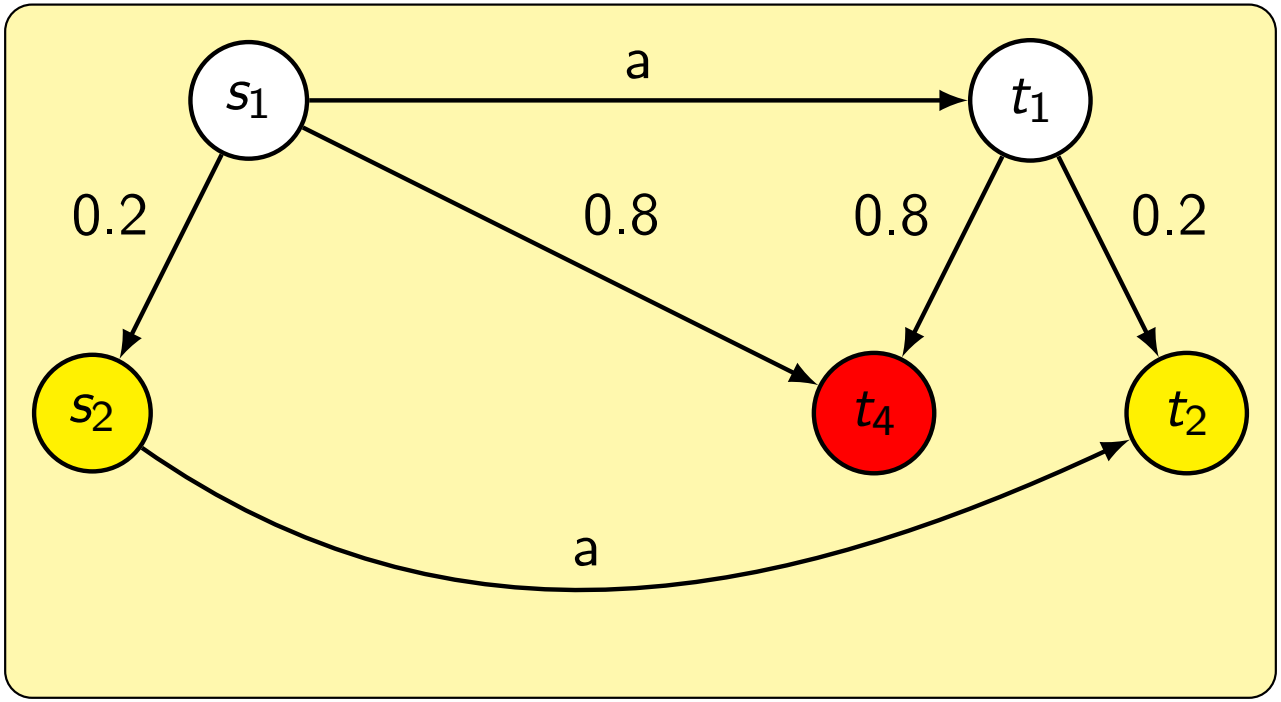

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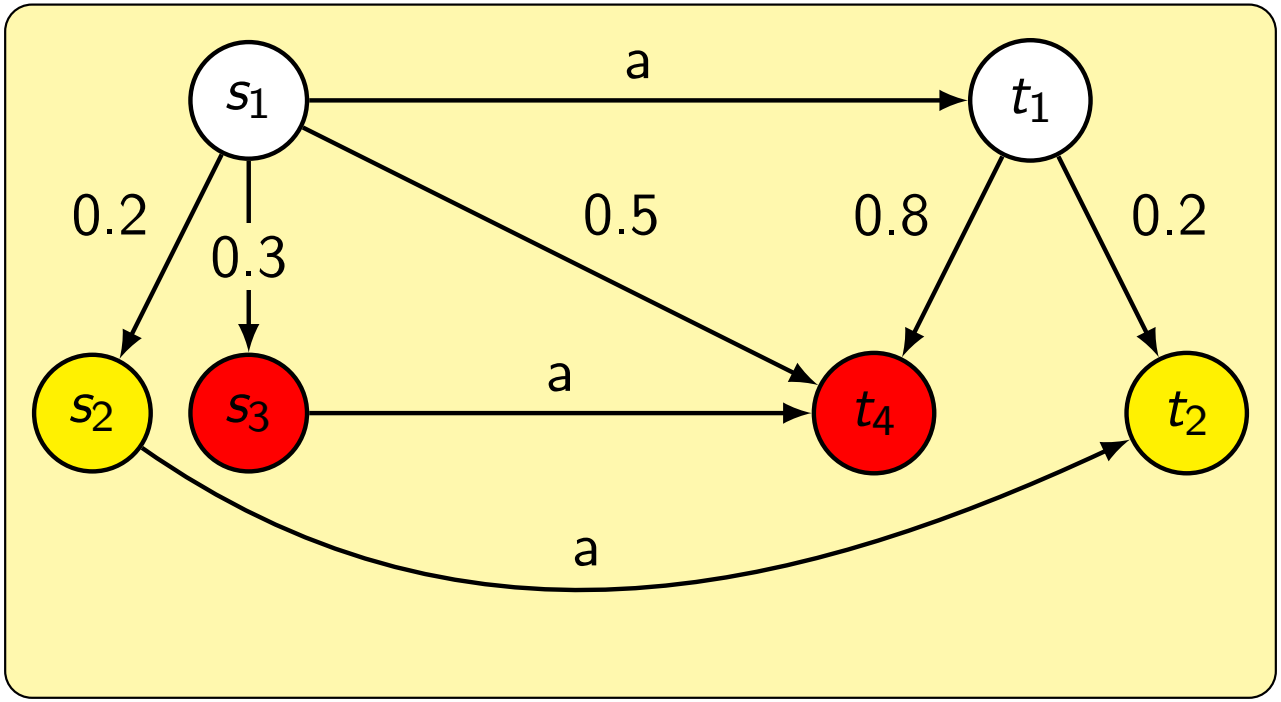

$$\mu(S_i) = \nu(s_i) \wedge (S_i = \{s_i\} \vee \forall s \in S_i \,.\, \exists a \in \Sigma \,.\, s \xrightarrow{a} s_i \in \mathcal{T})$$

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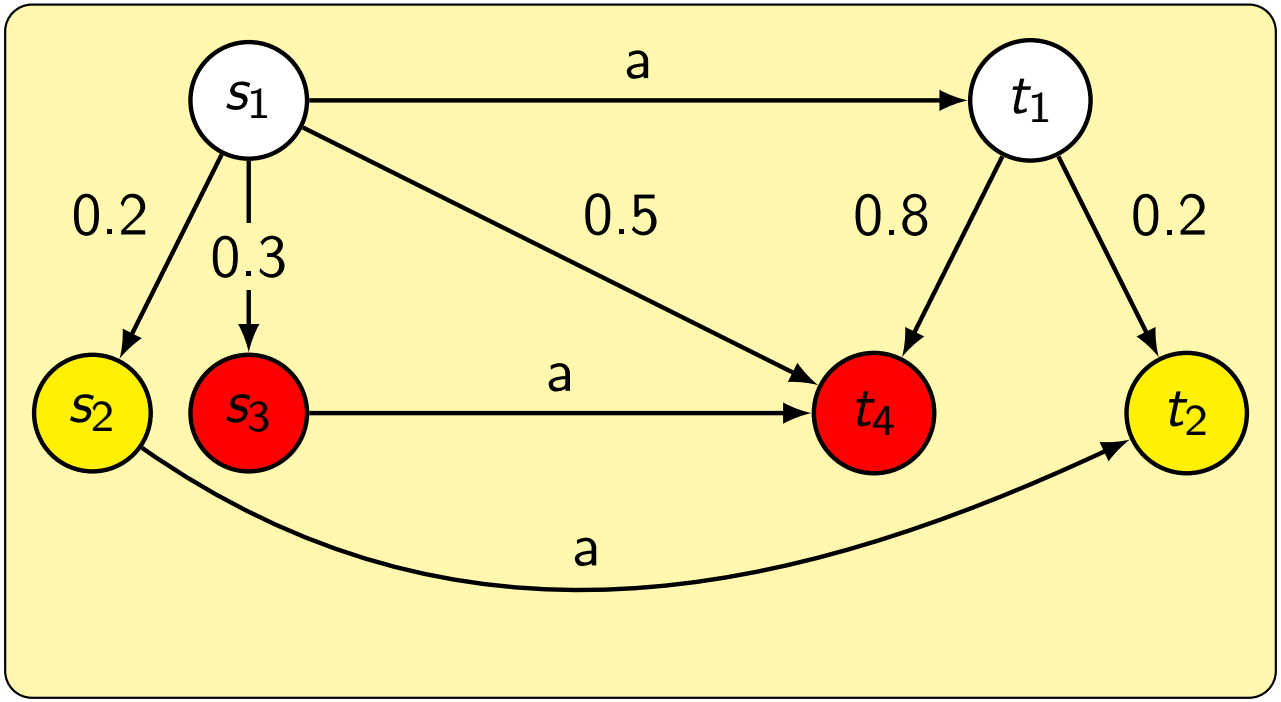

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$$\{(s,t) \mid s \in \mathrm{spt}(\mu), t \in \mathrm{spt}(\nu), s \xrightarrow{a} t \in \mathcal{T}\}$$

Correctness of confluence reduction

Even though

- Transitions may be mimicked by differently-labelled transitions
- Transitions only have to be invisible locally
- We have a more liberal notion of equivalence of distributions

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Still we find:

Theorem

Confluent transitions can be given priority, preserving $PCTL^*_X$.

On-the-fly detection of confluence

Simulation using on-the-fly confluence detection:

1. Simulate until reaching a nondeterministic choice

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To check if a transition is confluent:

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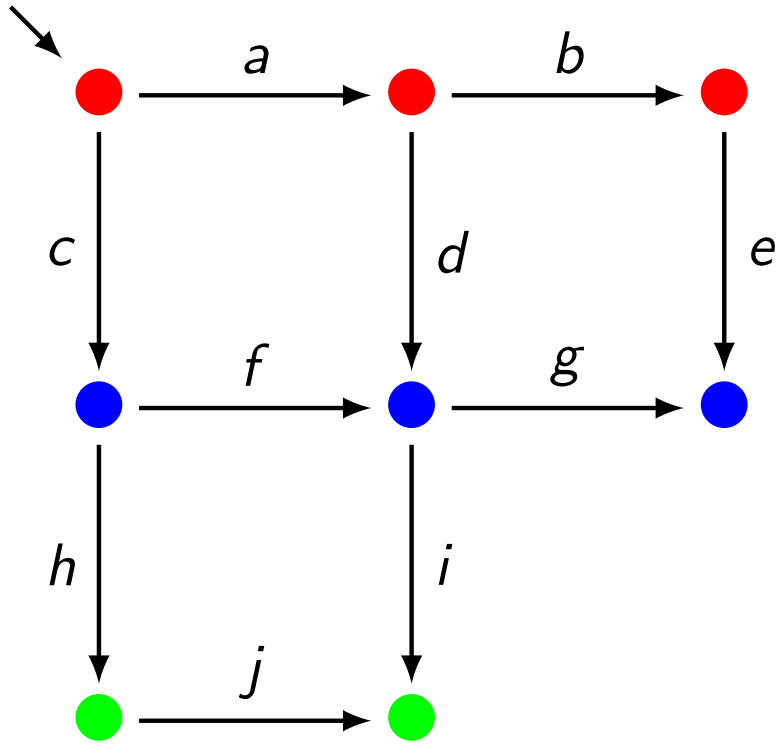
- Check if all its neighbouring transitions are mimicked
  - For this, additional transitions might need to be confluent

Checking a transition for confluence



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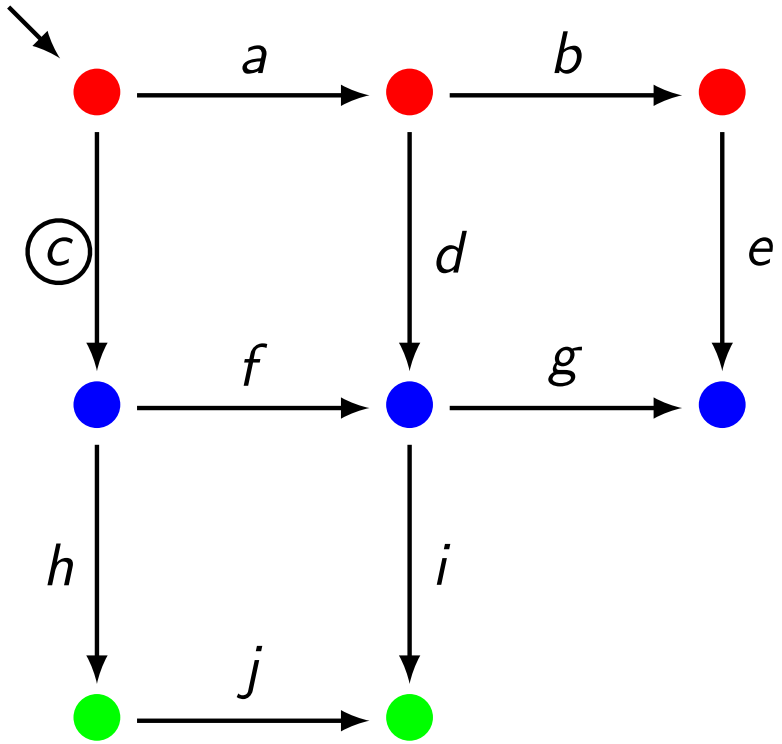

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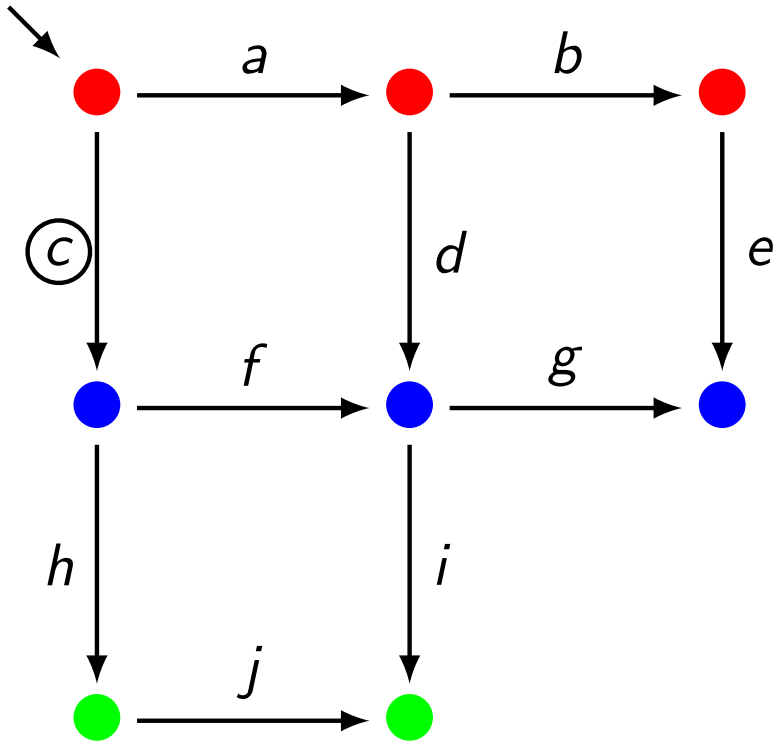

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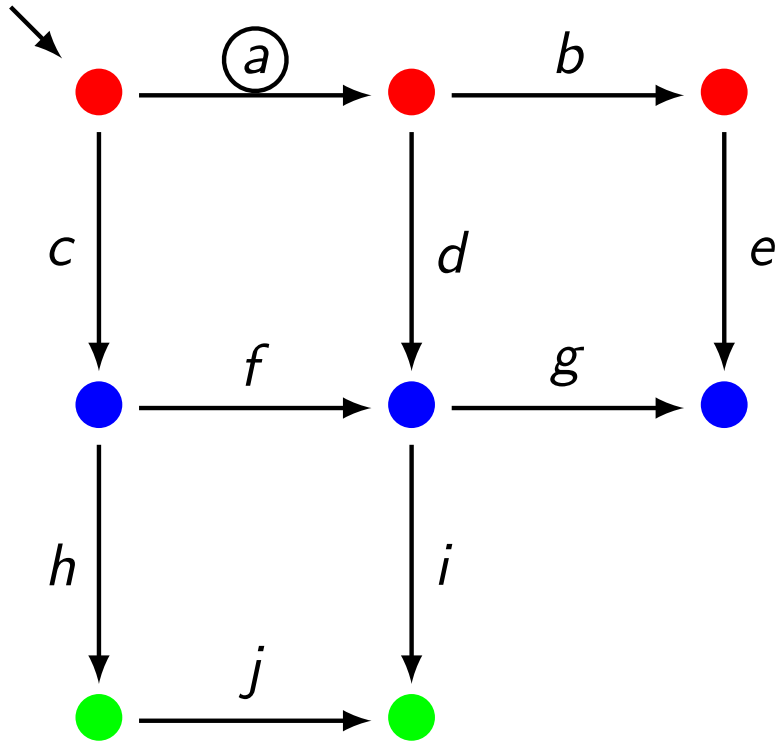

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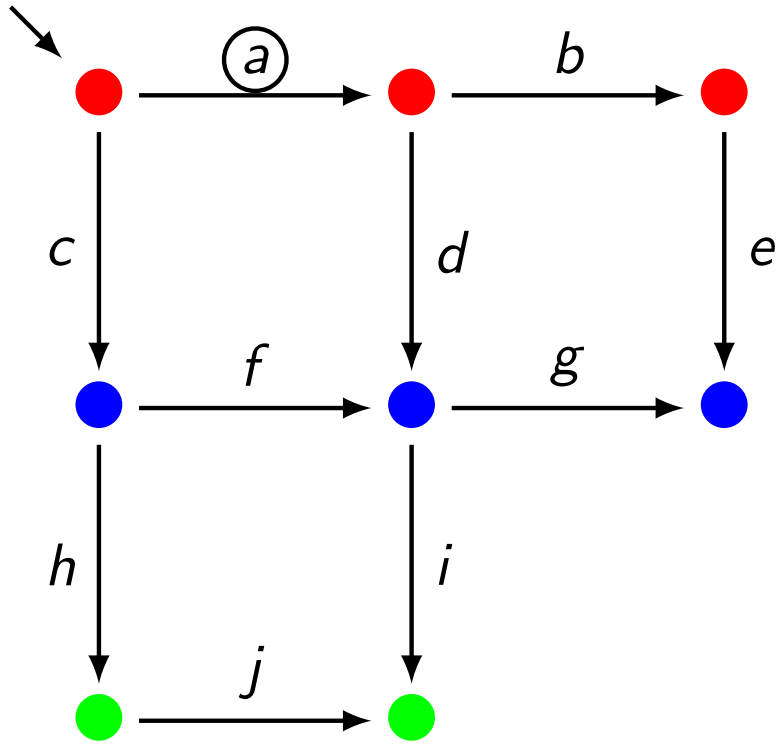

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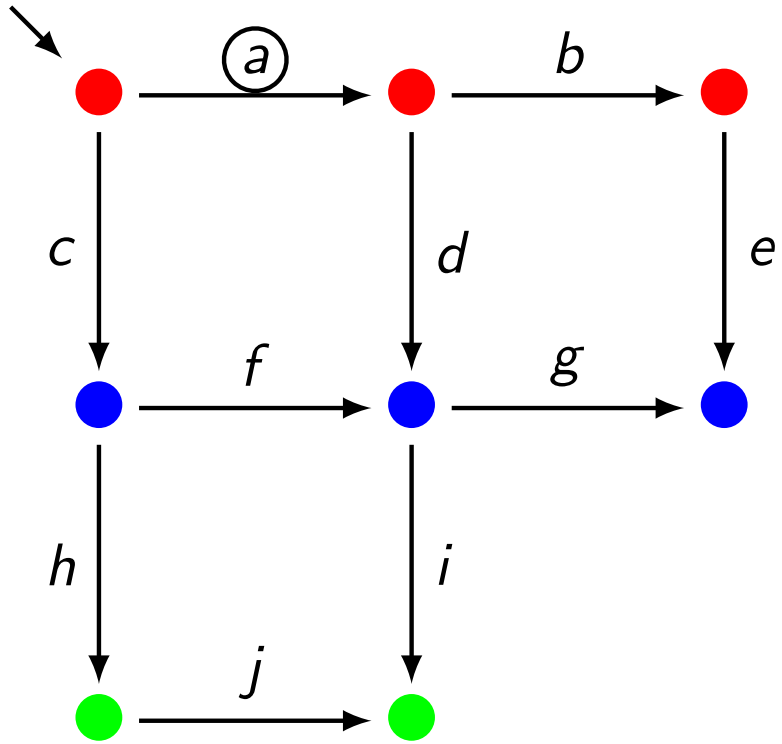

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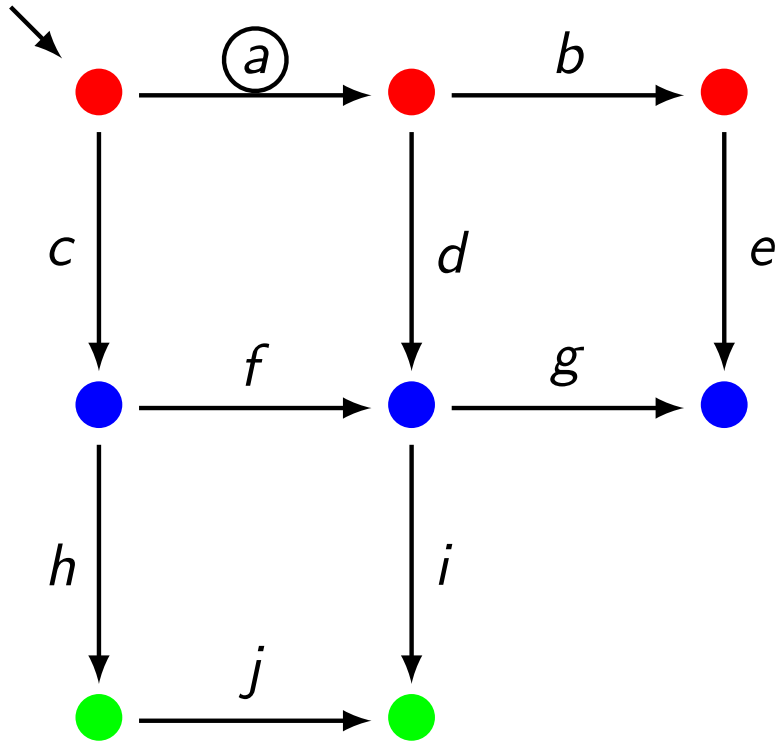

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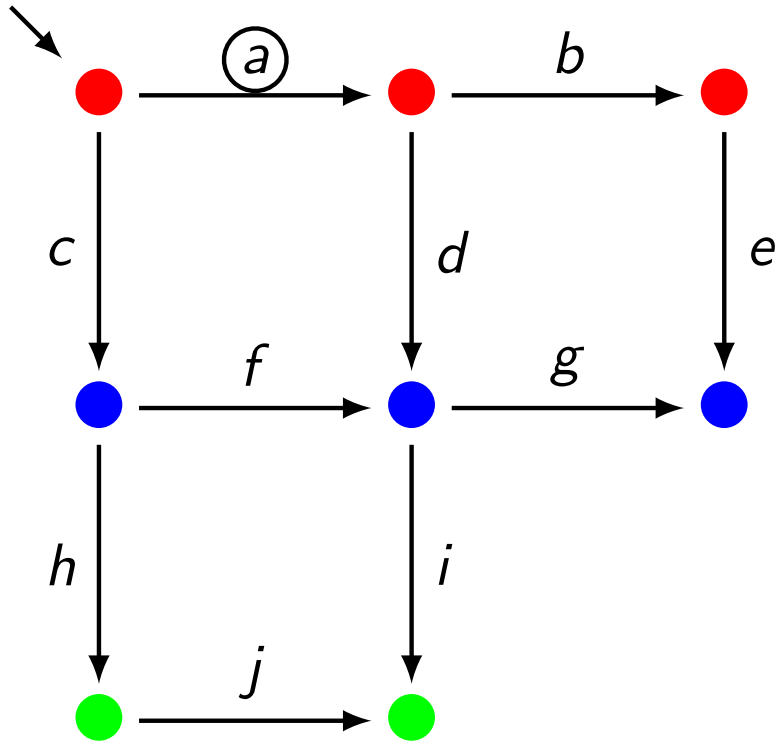

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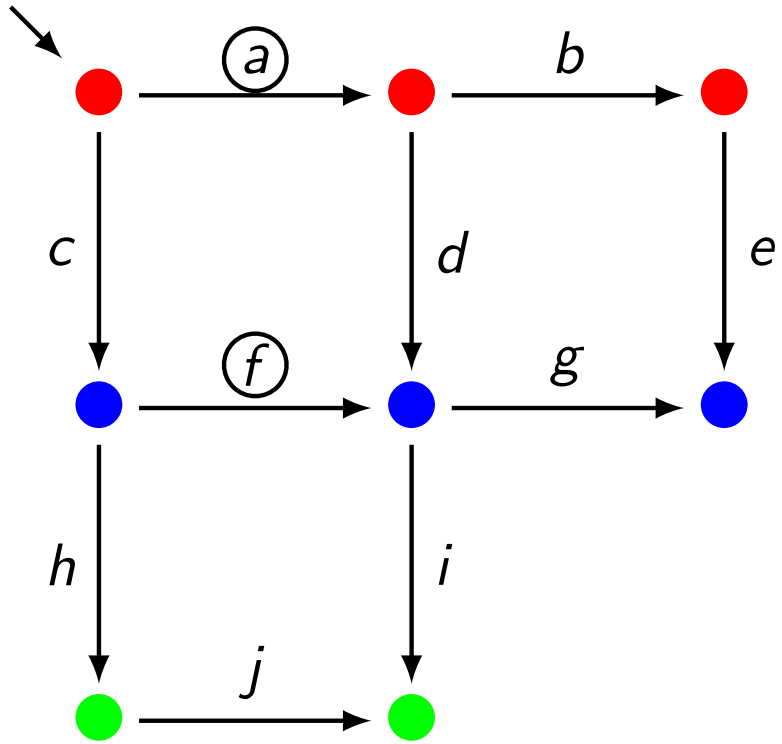

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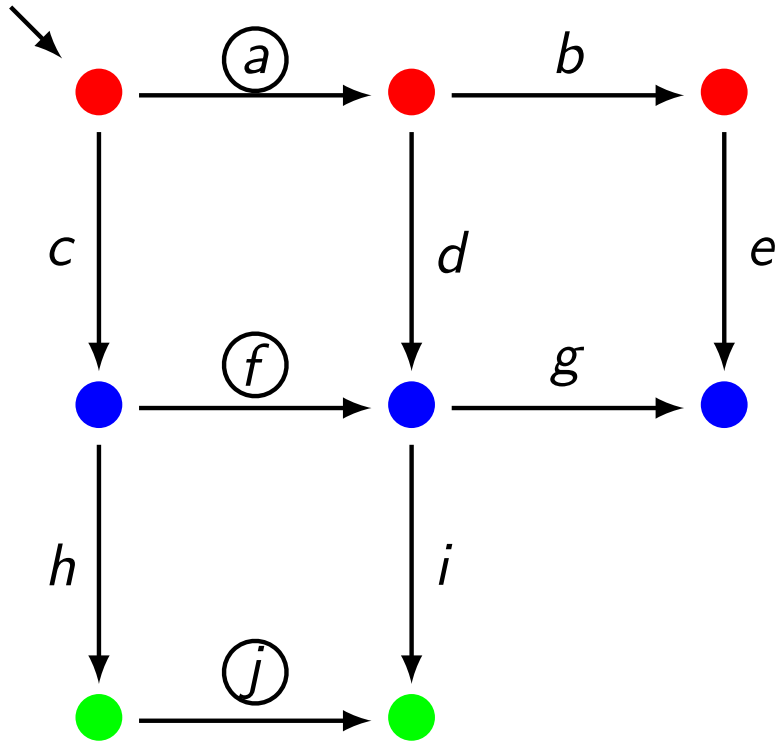

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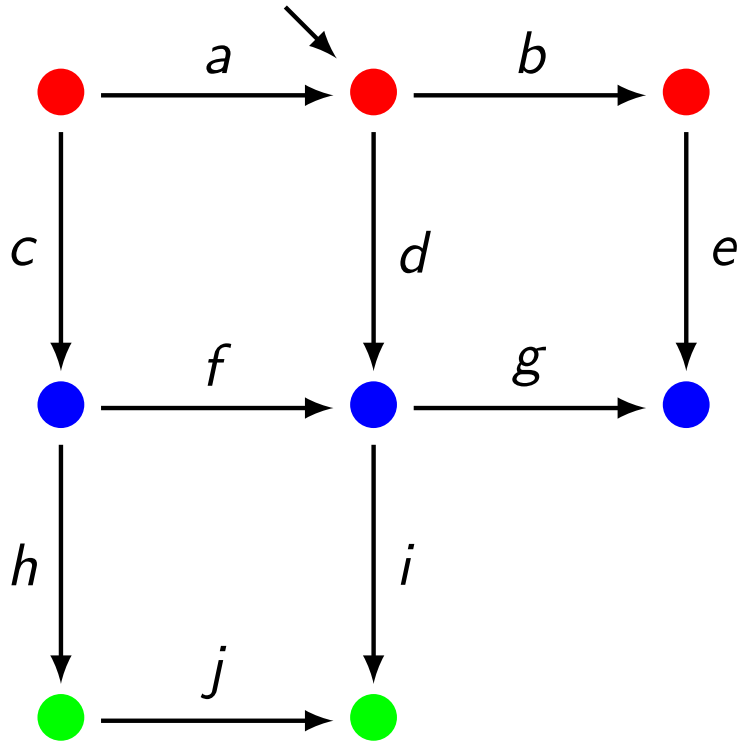

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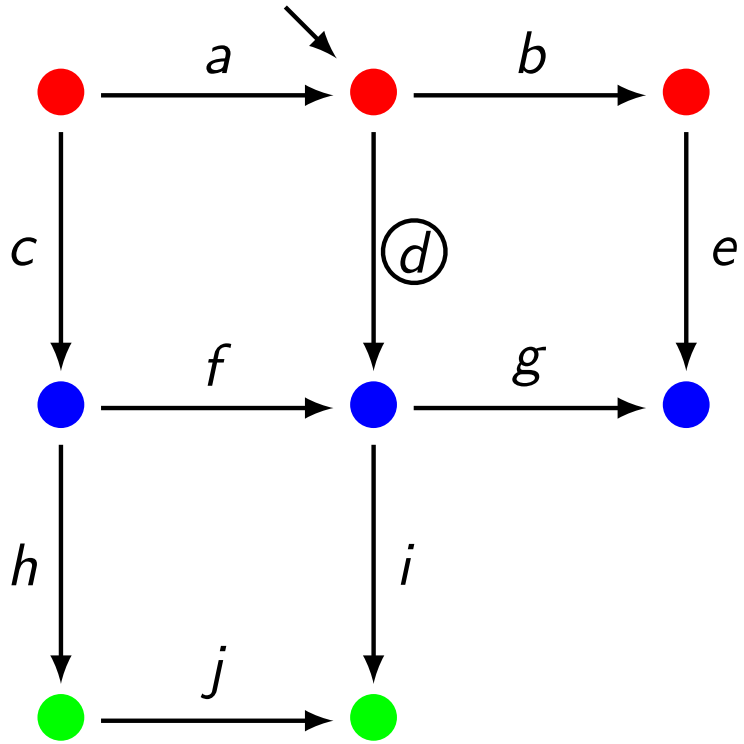

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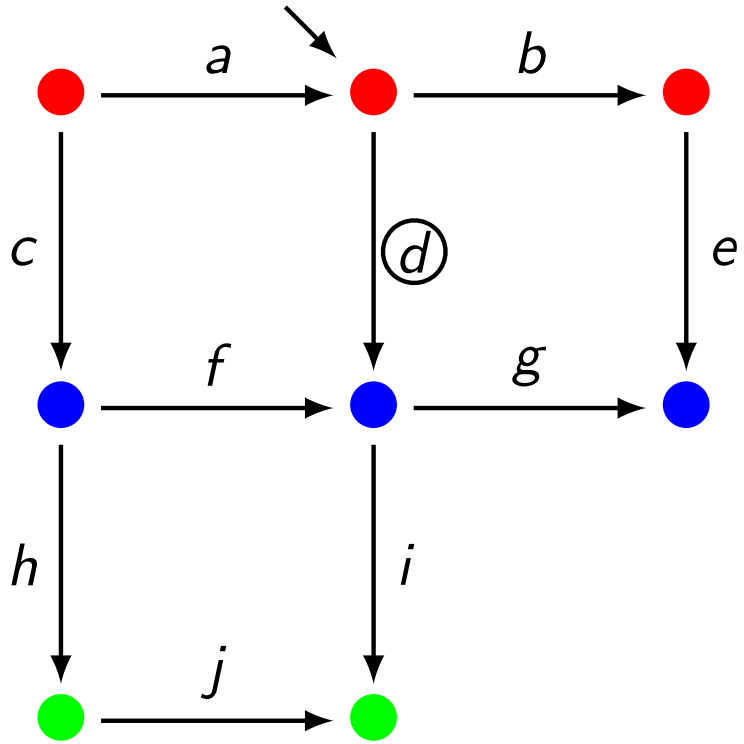

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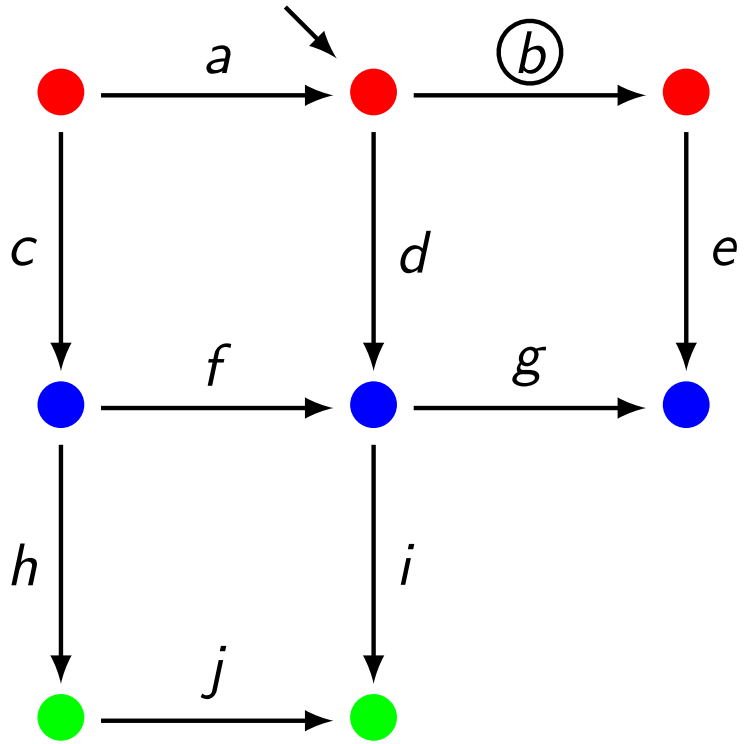

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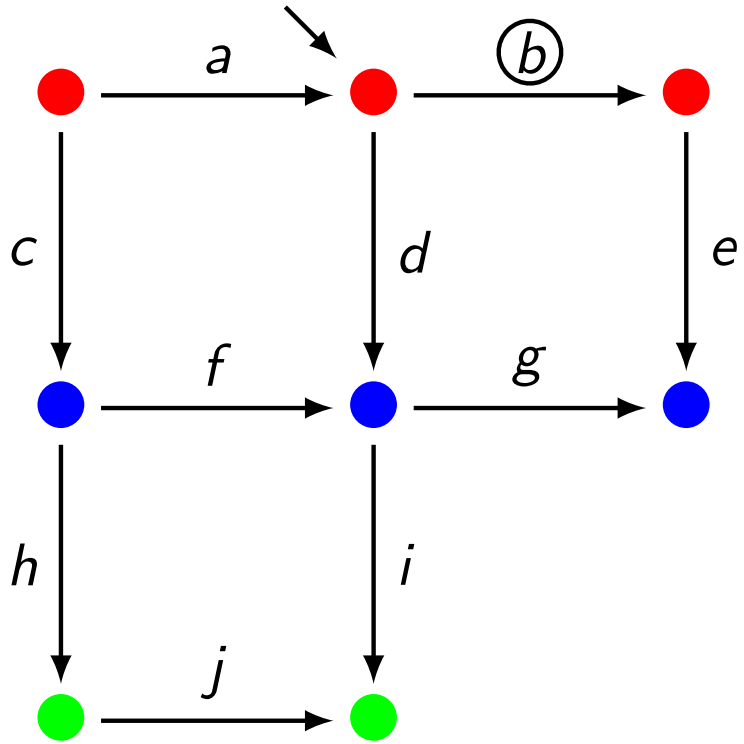

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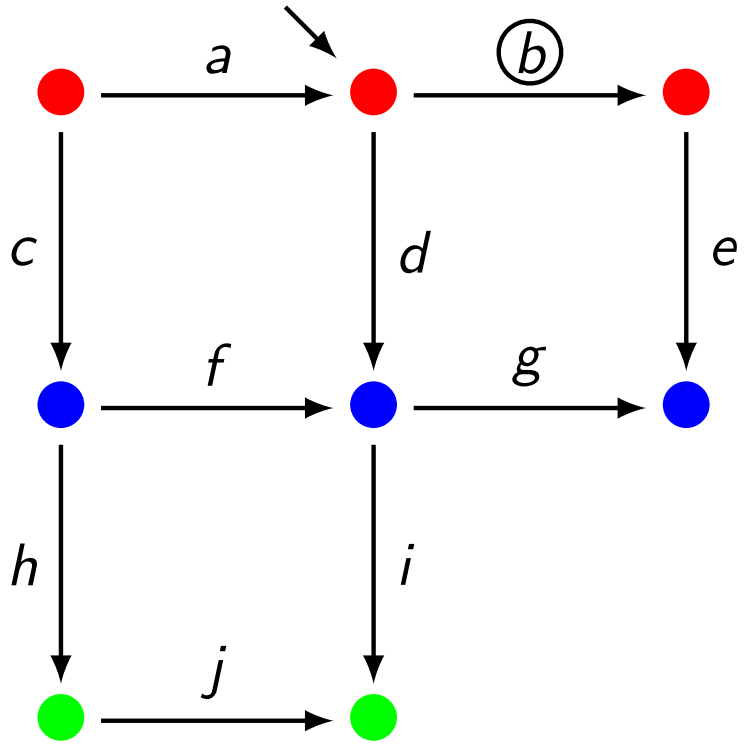

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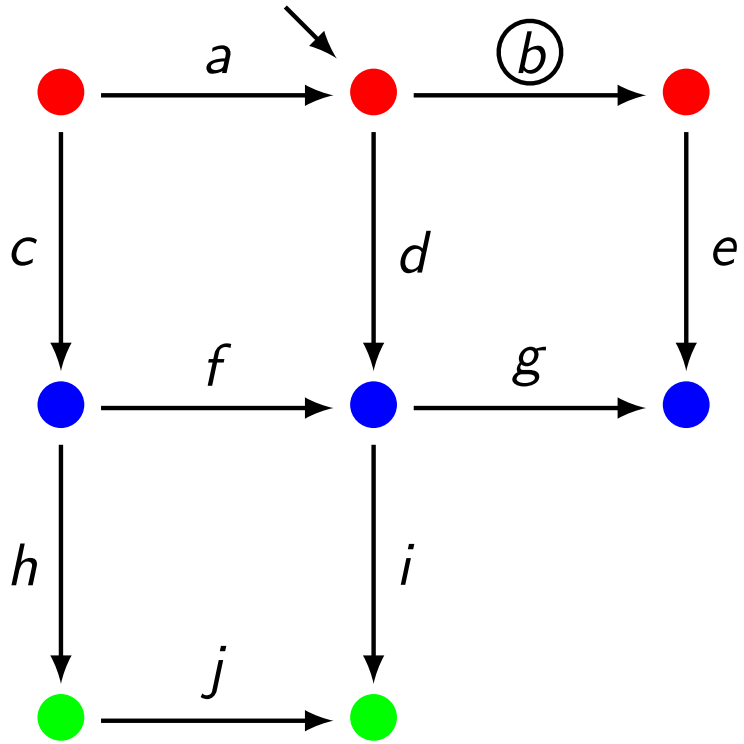

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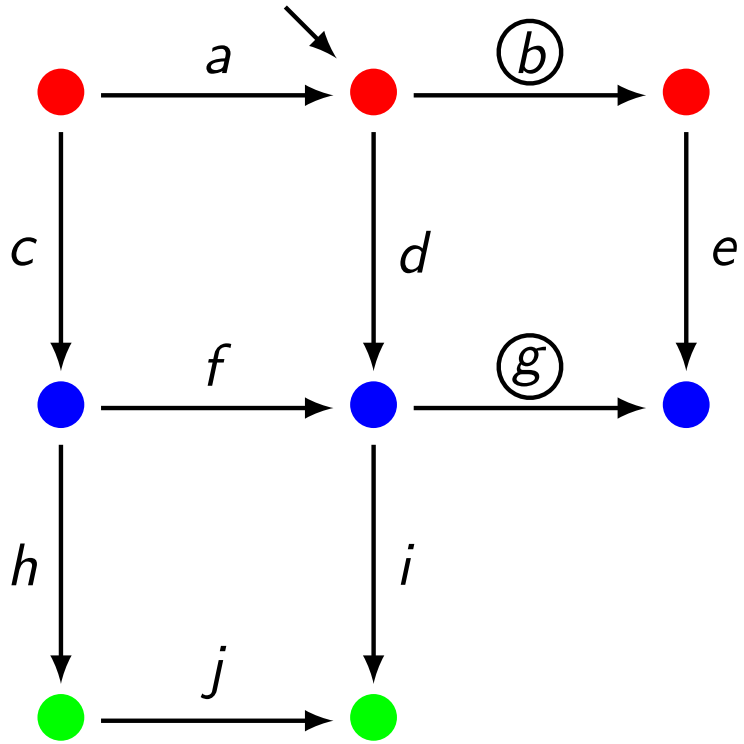

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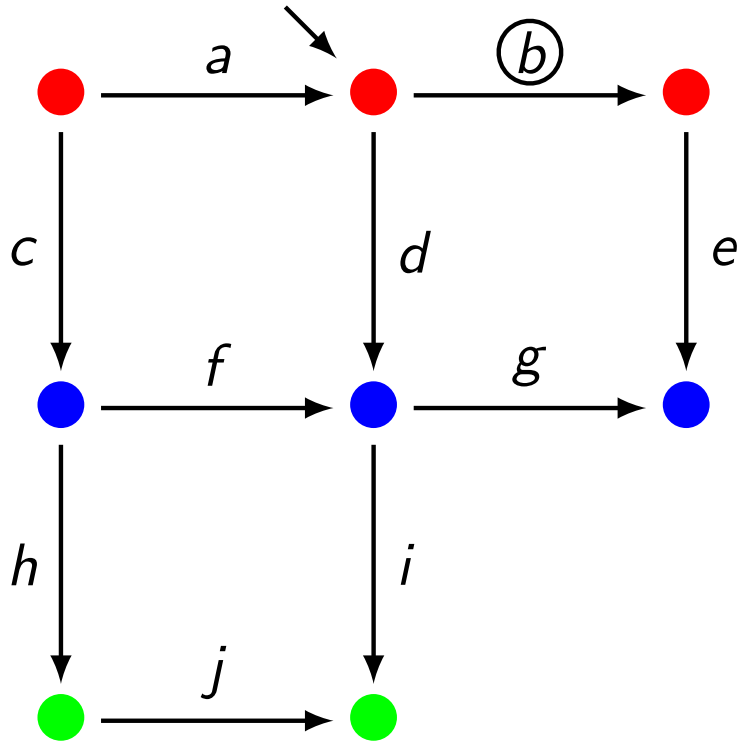

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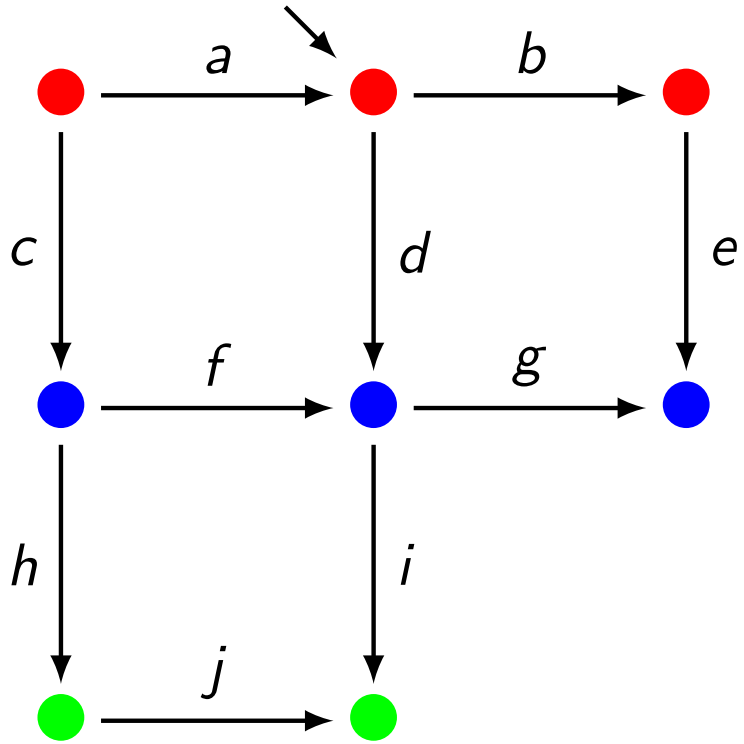

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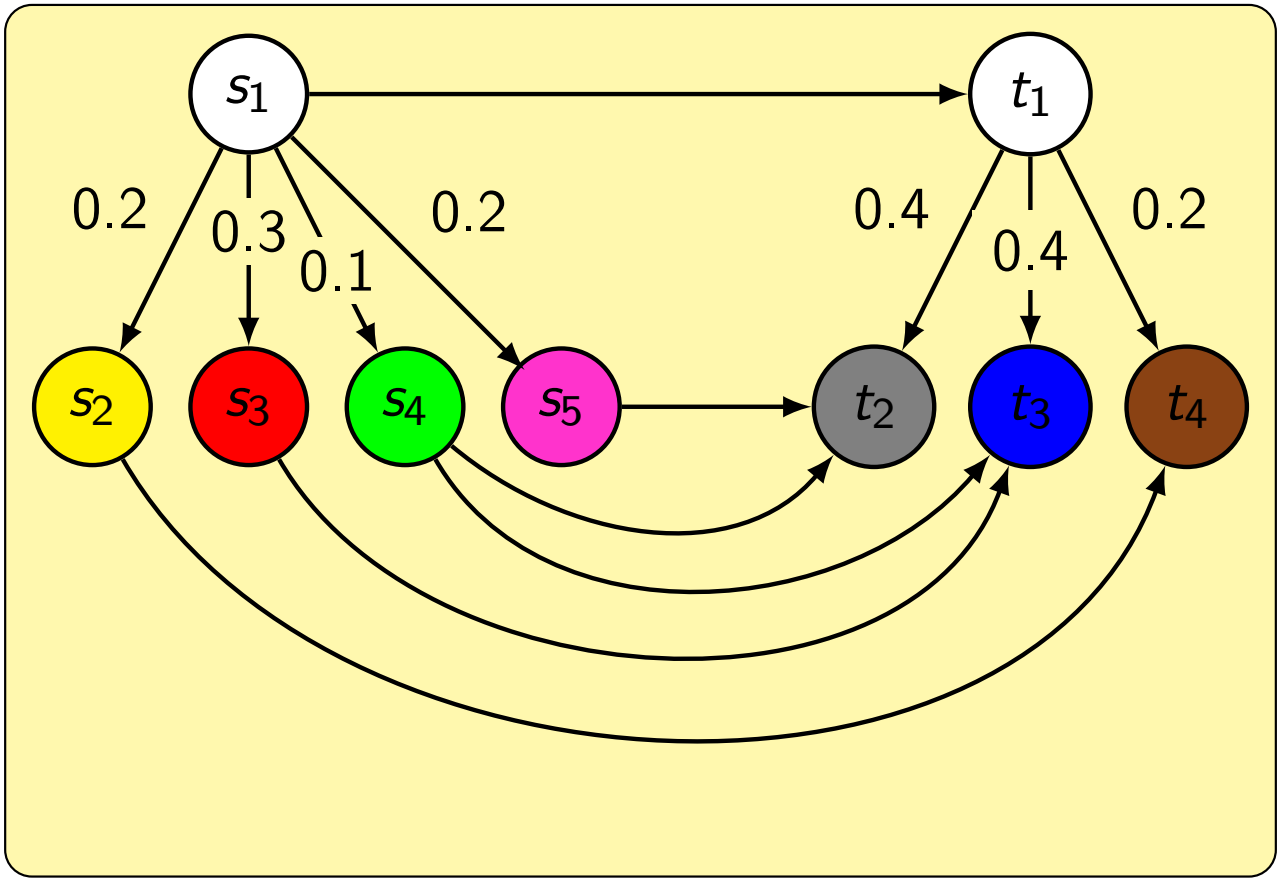

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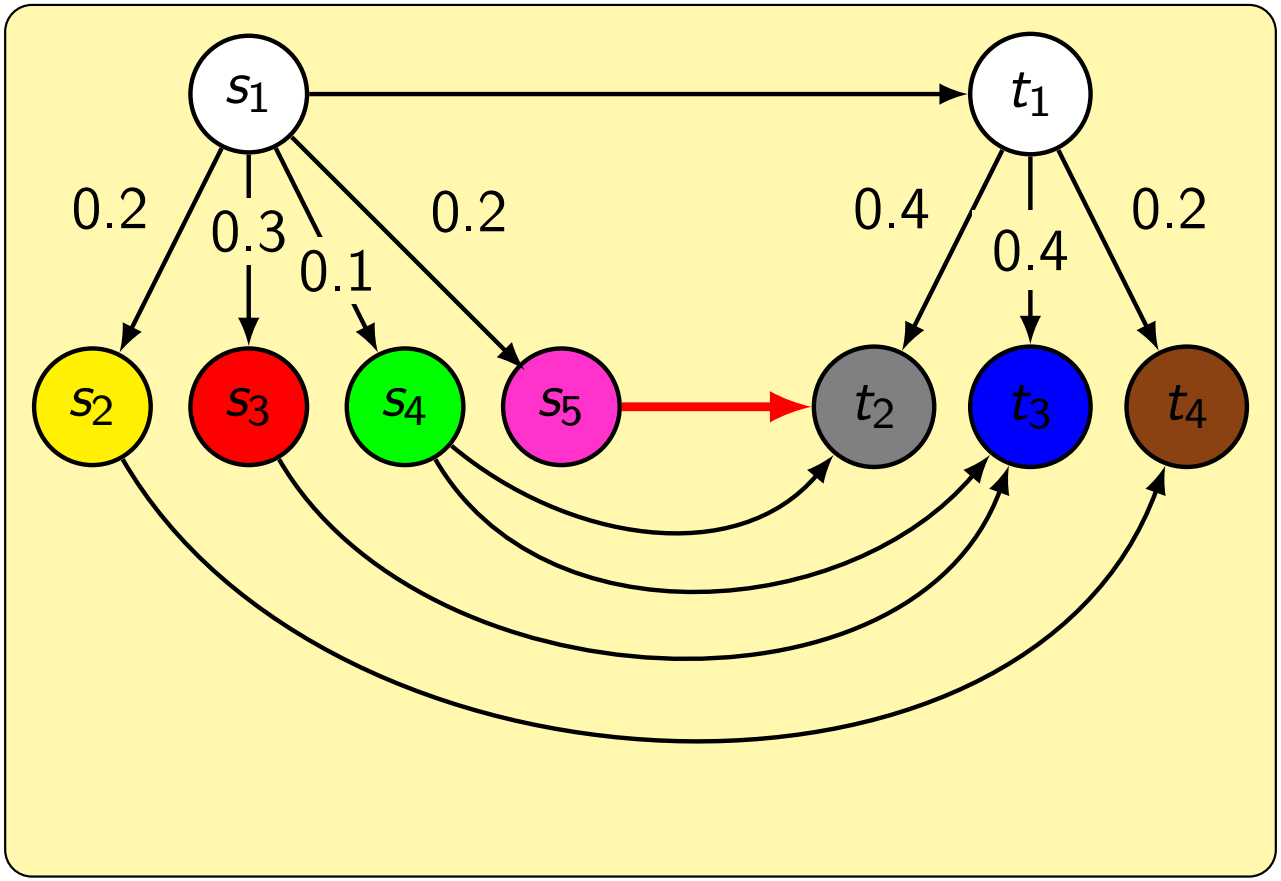

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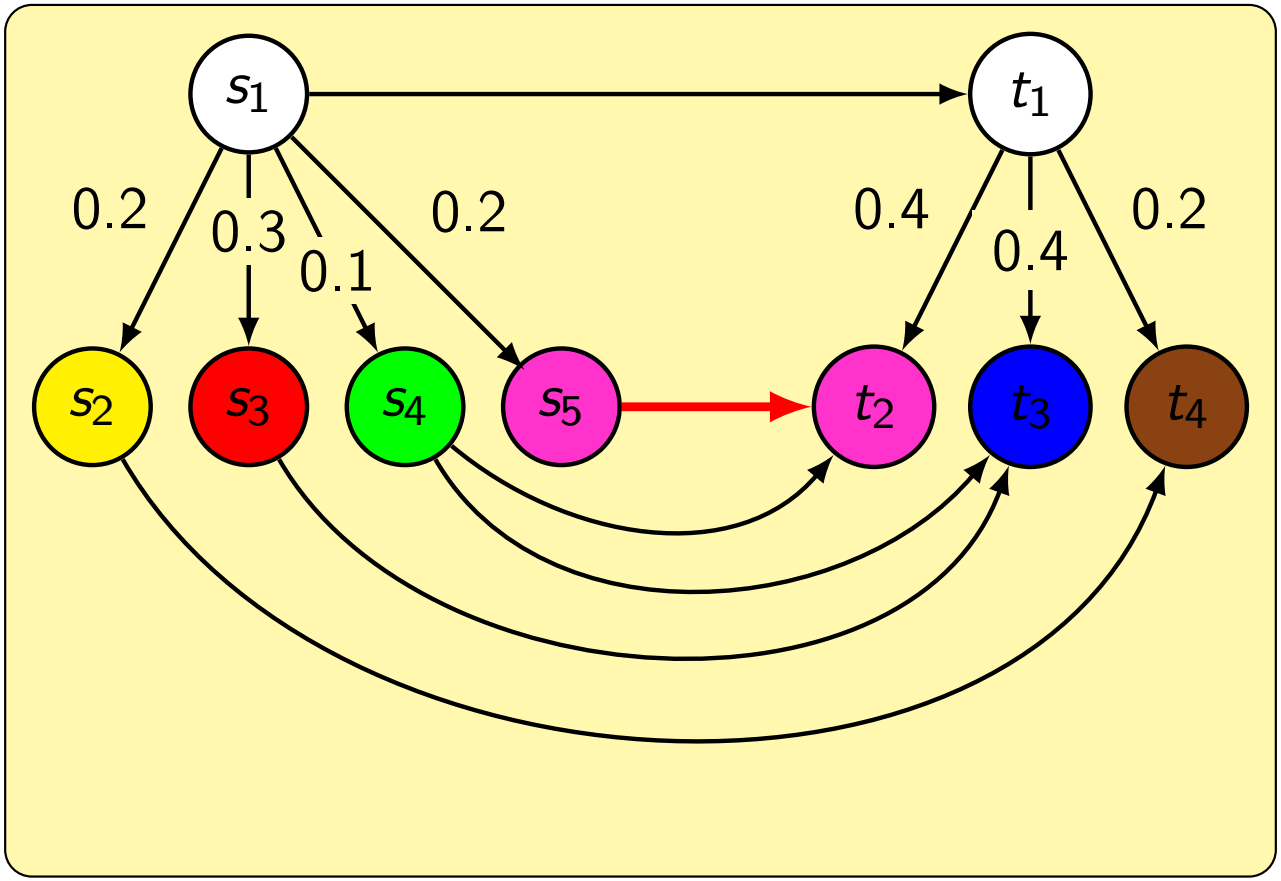

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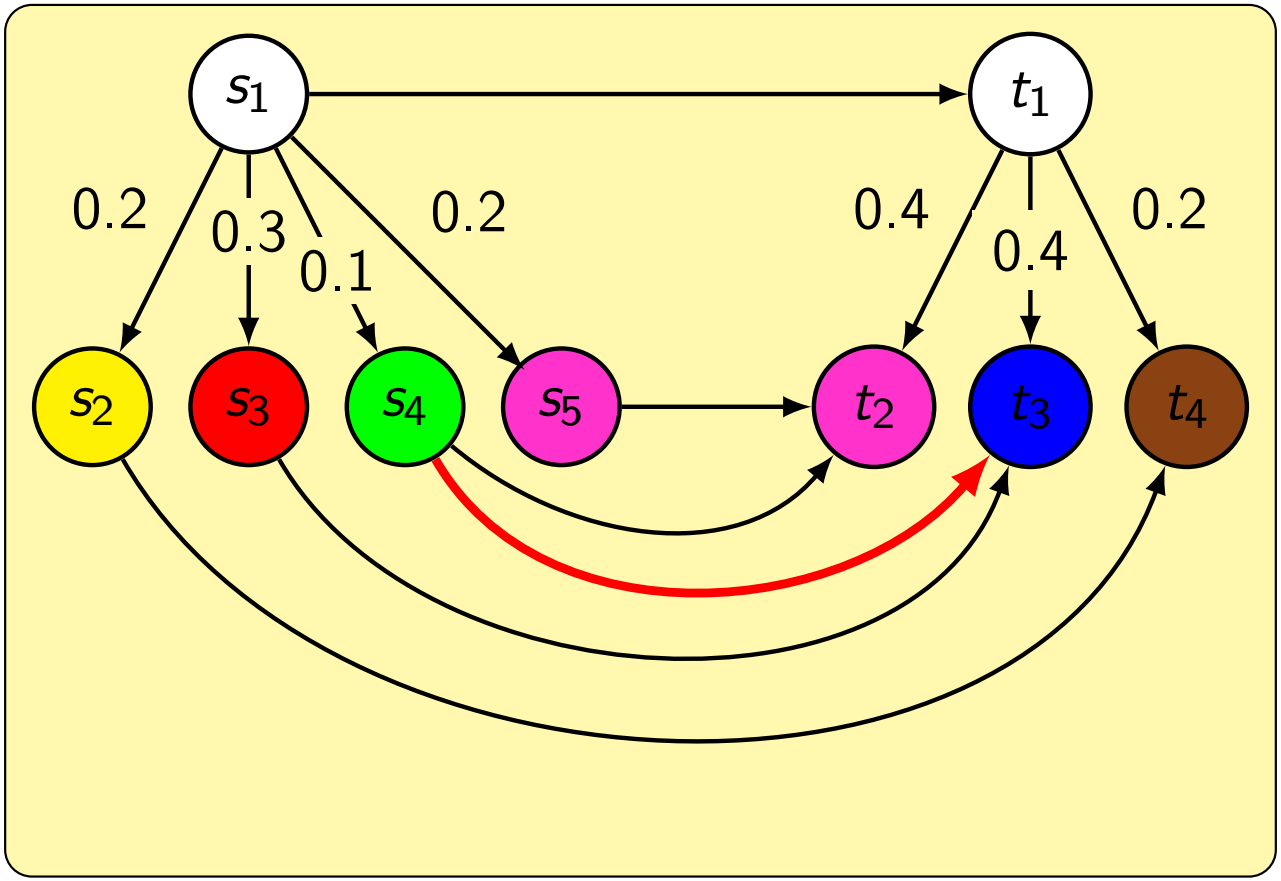

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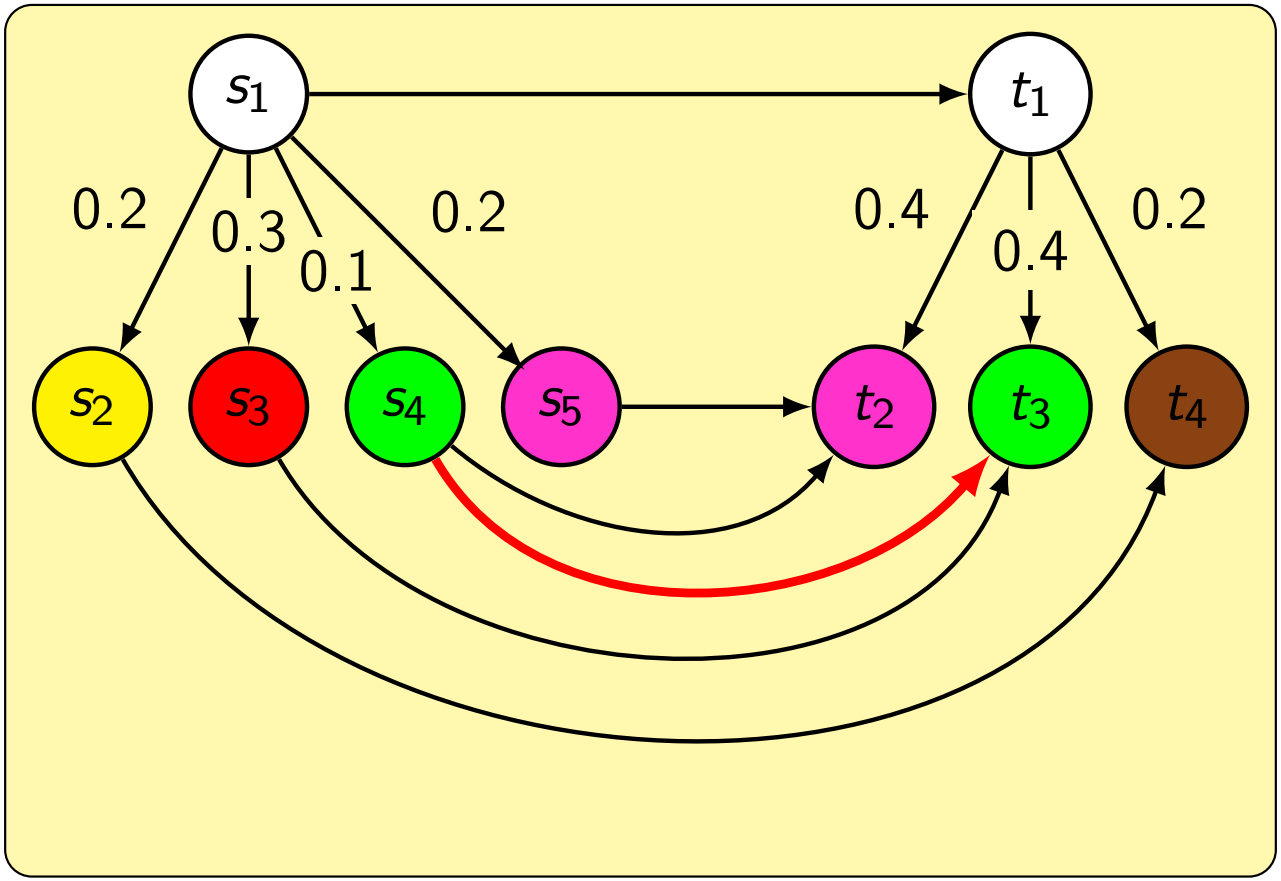

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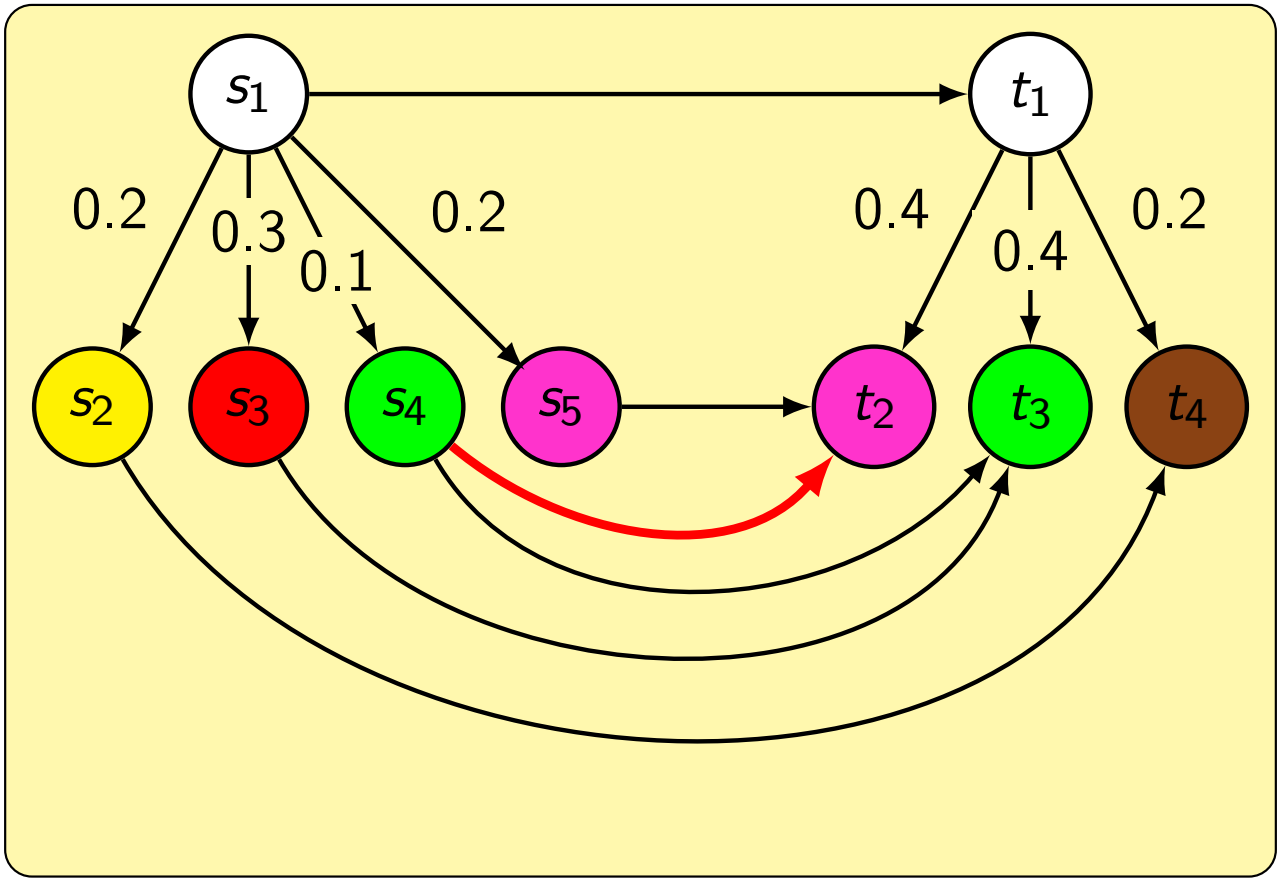

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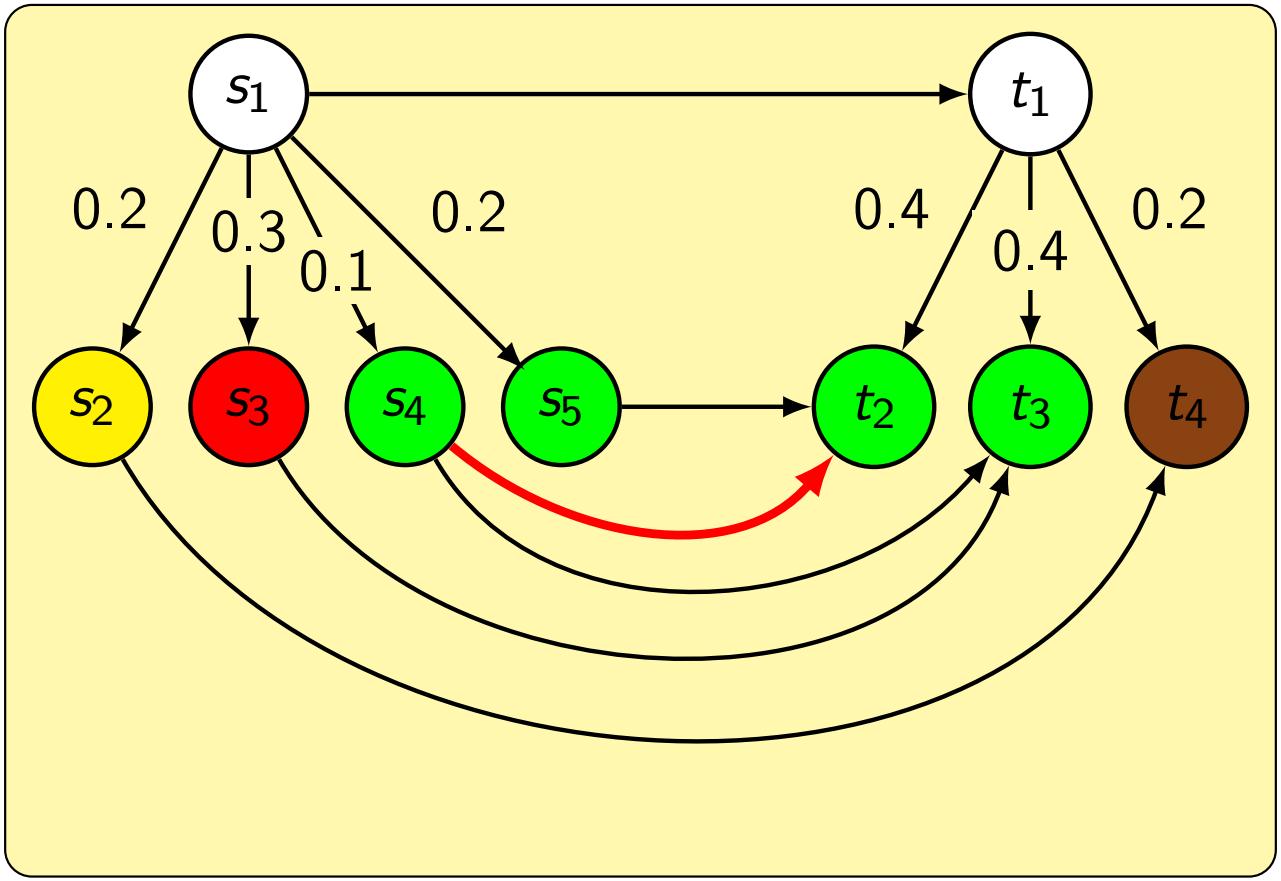

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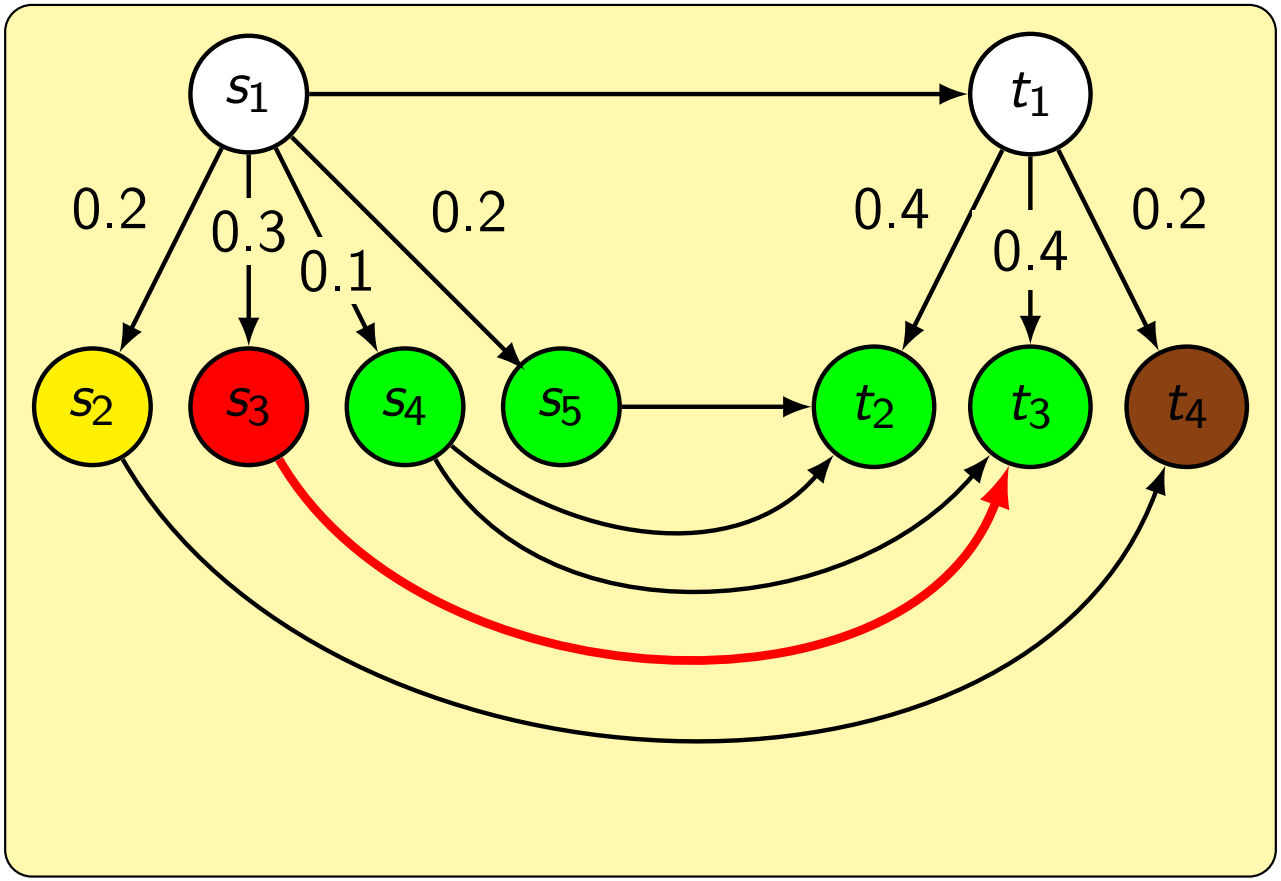

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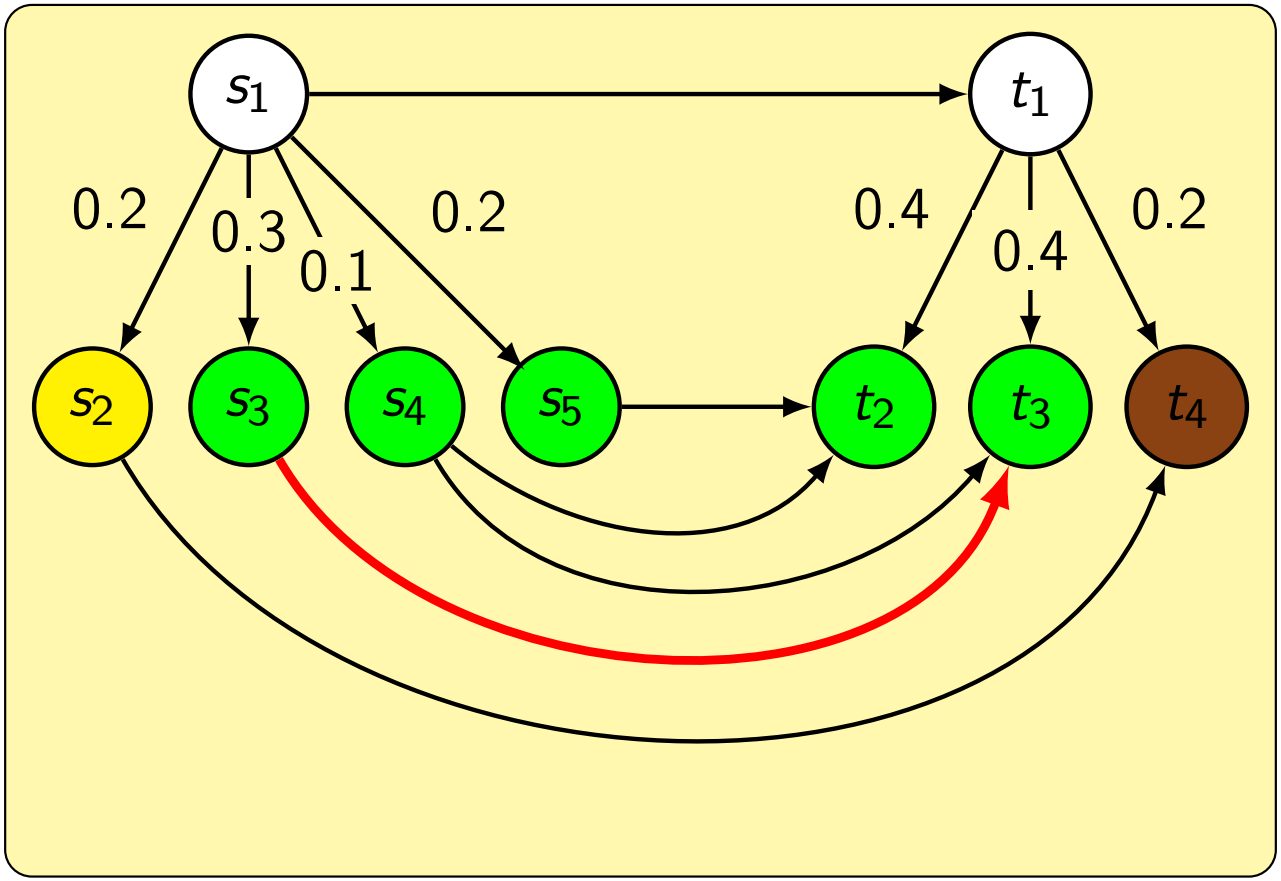

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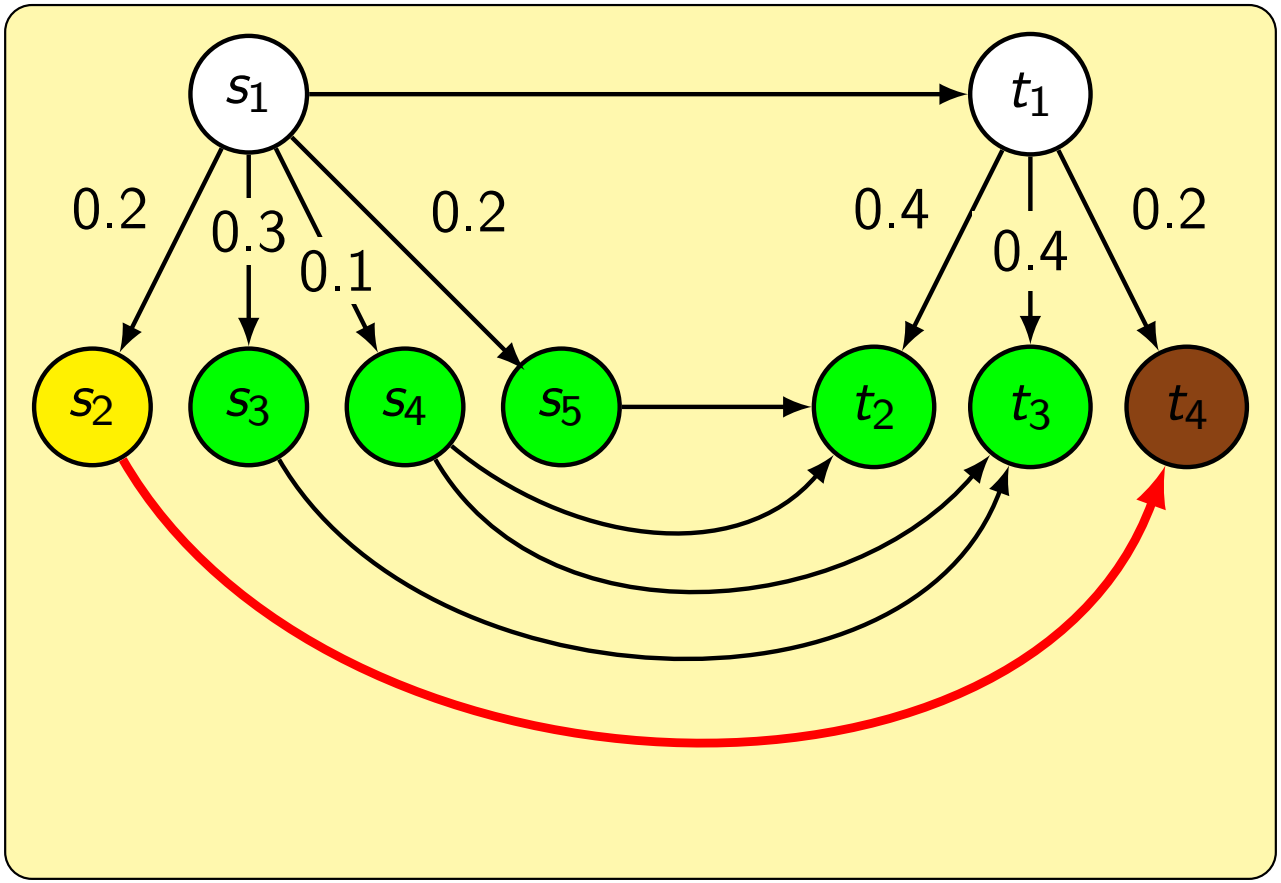

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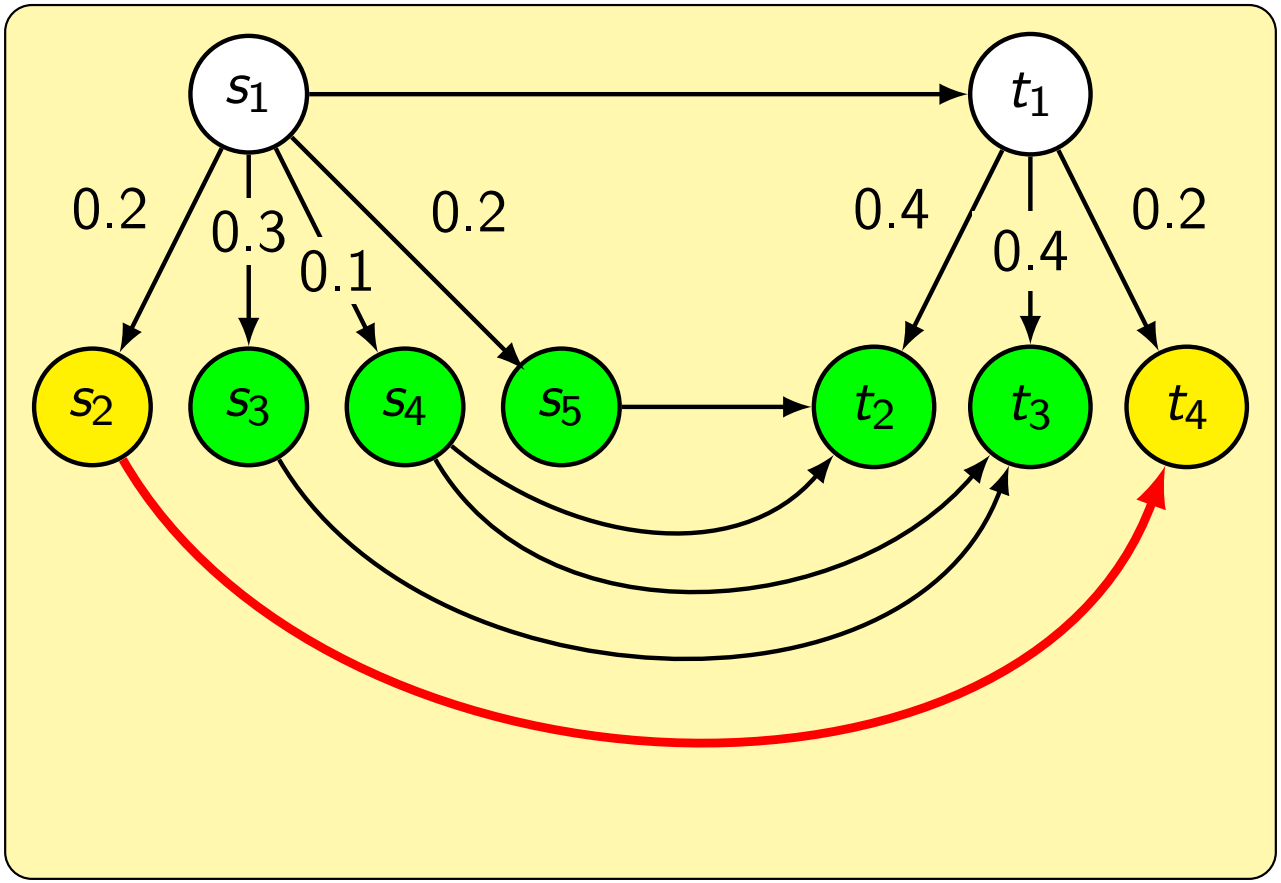

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Implementation

modes: a discrete-event simulator for the MODEST language

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  - Partial order reduction
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Three case studies:

- Dining Cryptographers
- IEEE 802.3 CSMA/CD
- Binary Exponential Backoff

Case study: Dining Cryptographers

Table : Confluence simulation runtime compared

model	simulation	
(N)	uniform	confluence
3	3 s	13 s
4	4 s	66 s
5	5 s	338 s

Partial order reduction was not able to resolve the nondetermism.

Case study: CSMA/CD

Table : Confluence simulation runtime compared

model	simulation		model checking	
(RED, BC_{MAX})	uniform	confluence	states	time
$(2, 1)$	6 s	18 s	15283	11 s
$(1, 1)$	6 s	18 s	30256	51 s
$(1, 2)$	11 s	48 s	194818	214 s

Partial order reduction was not able to resolve the nondetermism.
(for confluence, probabilistic transitions needed to be synchronised)

Case study: Binary Exponential Backoff

Table : Confluence simulation runtime compared

model	simulation		
(K, N, H)	uniform	partial order	confluence
$(4, 3, 3)$	1 s	2 s	2 s
$(8, 7, 4)$	14 s	18 s	16 s

Conclusions

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  - Independent of action labels
  - Independent of global behaviour
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  - Independent of action labels
  - Independent of global behaviour
  - More liberal equivalence of distributions
- We provided an on-the-fly detection algorithm for SMC
- We implemented the new technique in MODEST
- Case studies show that confluence reduction reduces more and slightly faster than partial order reduction
- More models can now statistically be checked

Questions

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