UNIVERSITY OF TWENTE.

Formal Methods & Tools.

Confluence Reduction for Markov Automata

Mark Timmer

April 4, 2013

FMT Lunchmeeting

Joint work with
Jaco van de Pol and Mariëlle Stoelinga

The overall goal: efficient and expressive modelling

Specifying systems with

- Nondeterminism ⟵ LTSs
- Probability ⟵ DTMCs
- Stochastic timing ⟵ CTMCs

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(Nondeterminism and Probability →) Probabilistic Automata (PAs)

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Interactive Markov Chains (IMCs)

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Markov Automata (MAs)

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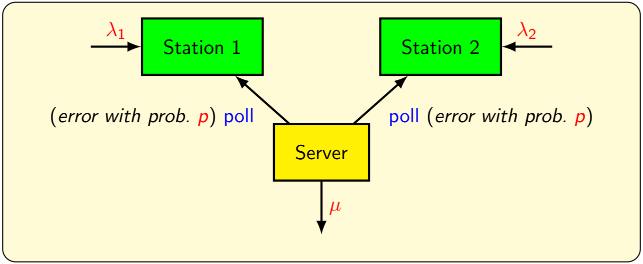
Markov Automata (MAs)



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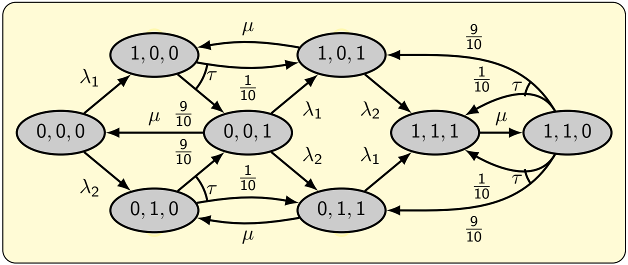

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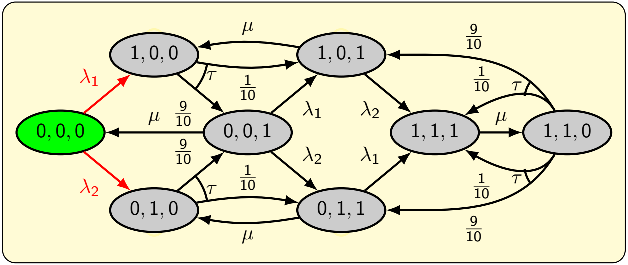

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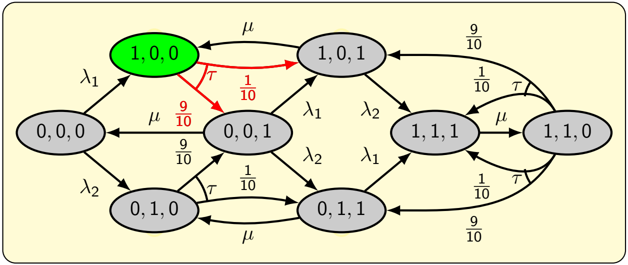

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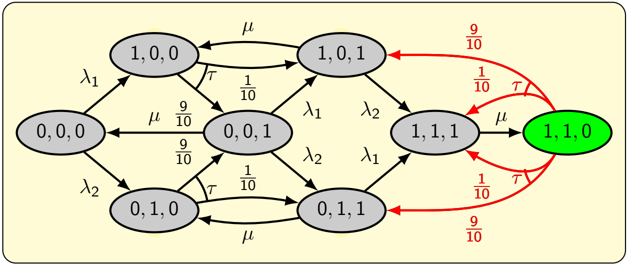
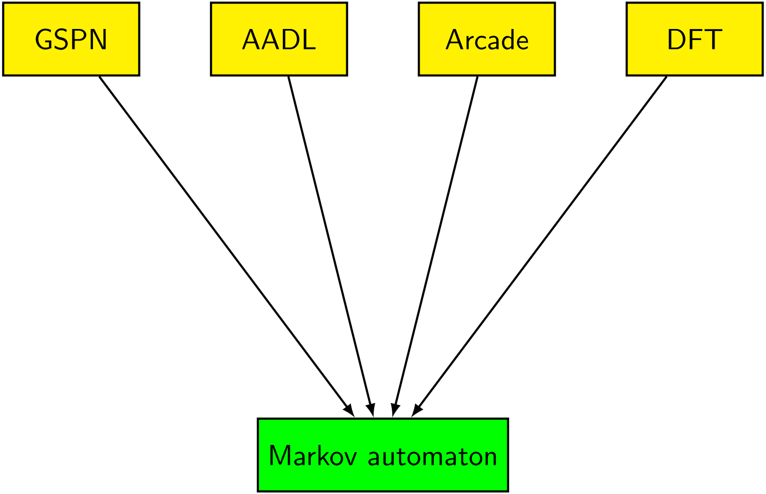


Higher-level formalisms that can be mapped to MAs



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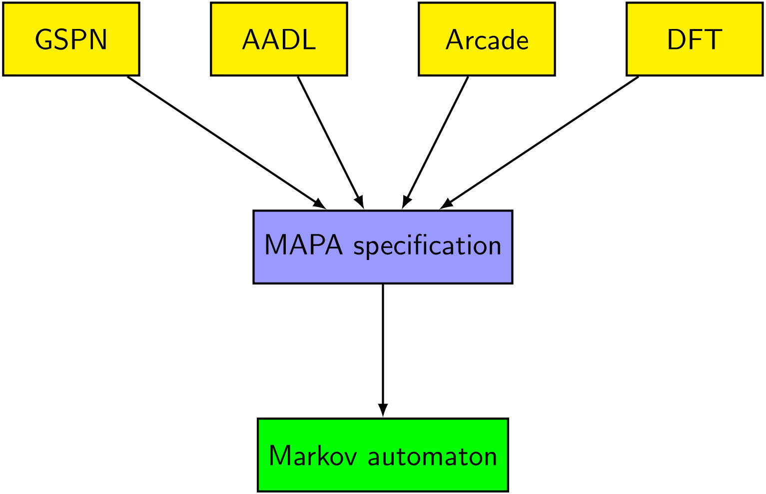

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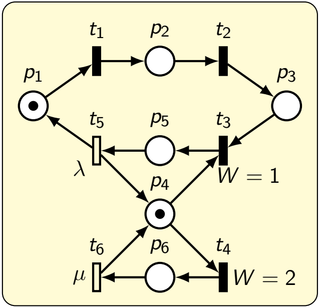

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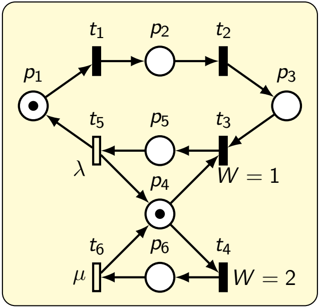

MAPA specification

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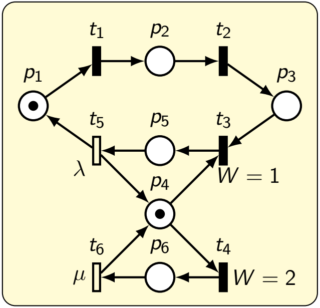

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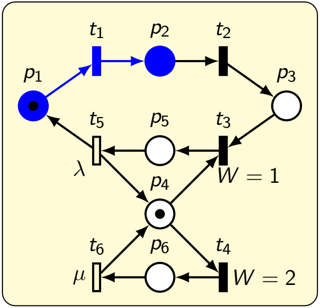

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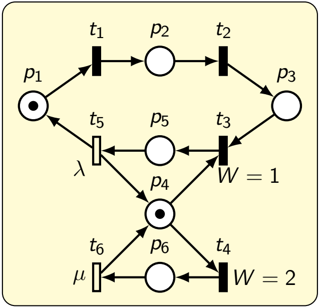

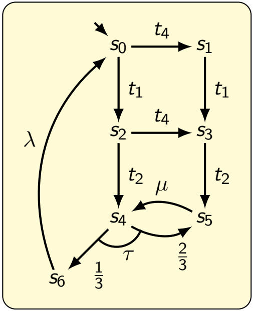

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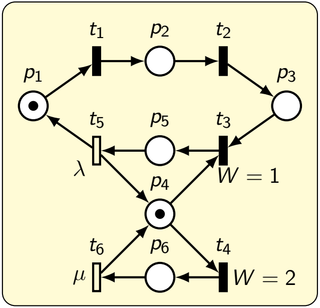

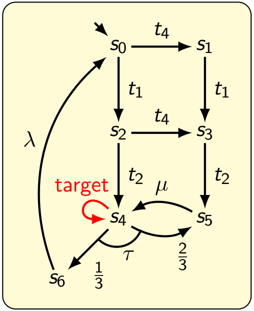

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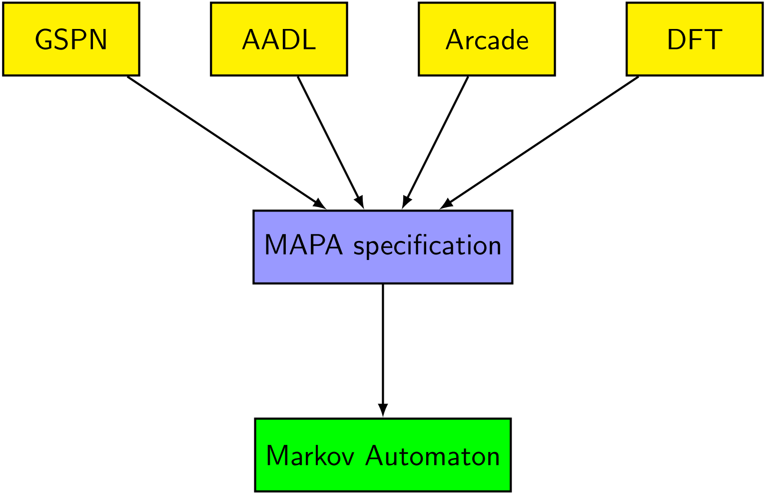

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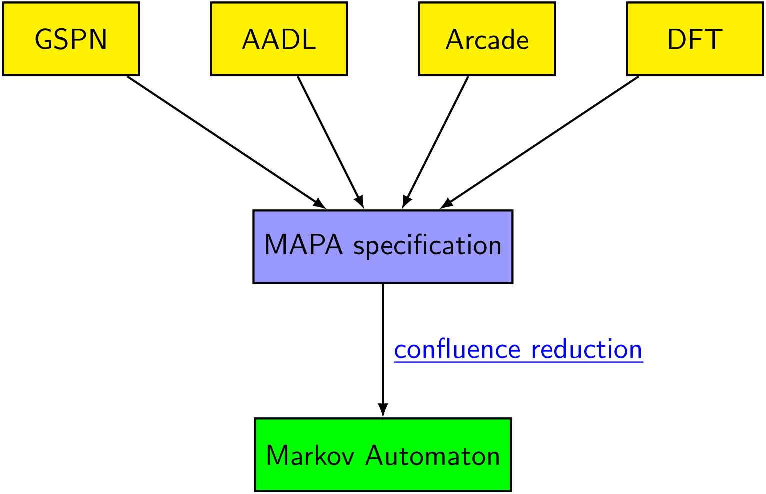

Contents

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Invisible transitions in confluence reduction:

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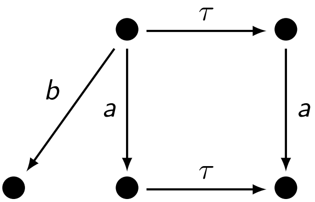

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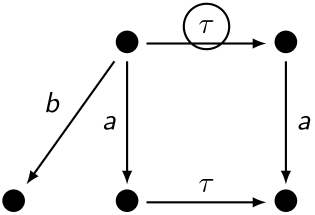

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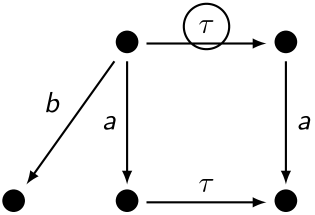

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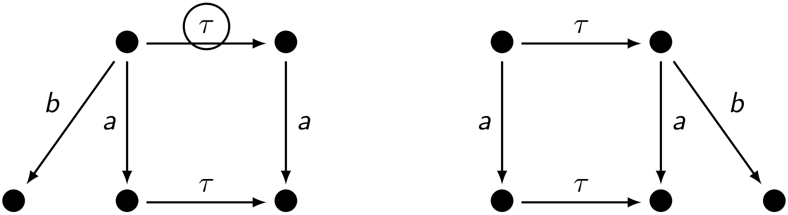

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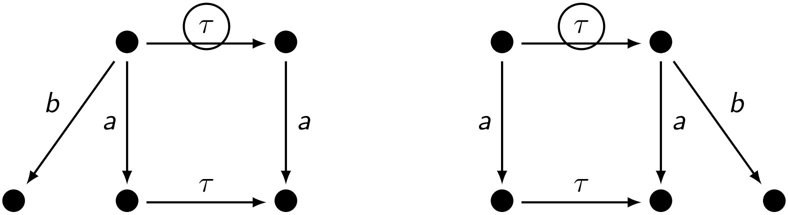

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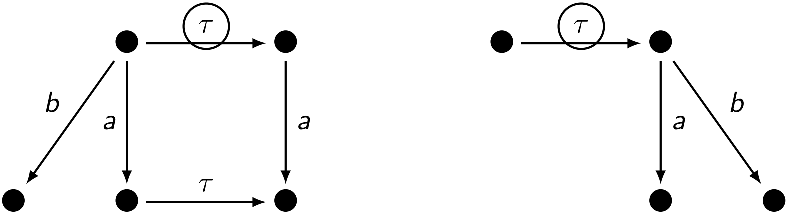

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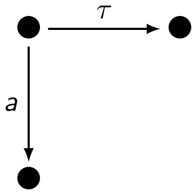

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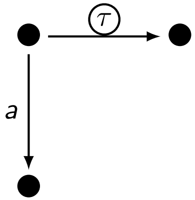

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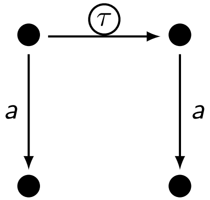

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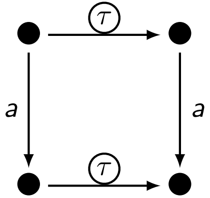

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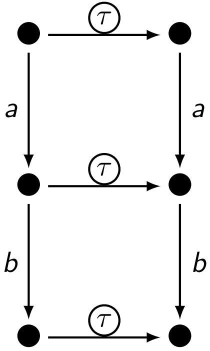

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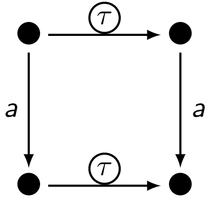

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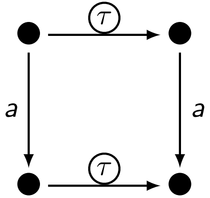

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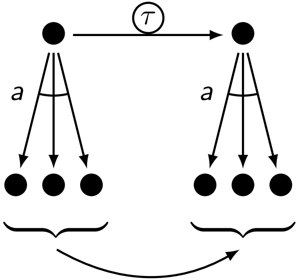

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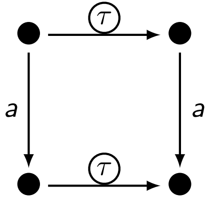

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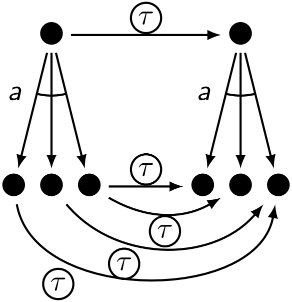

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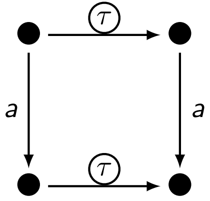

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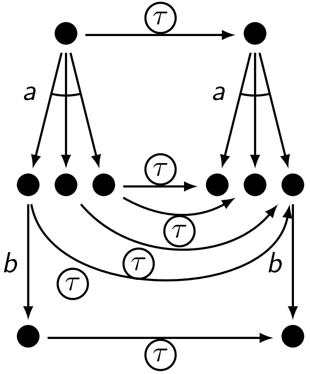
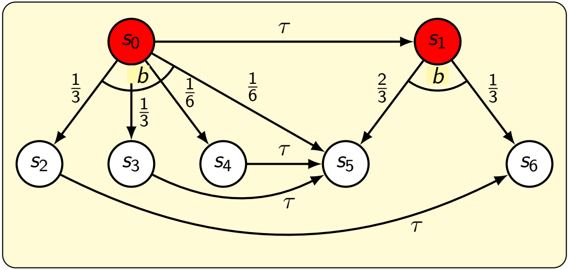


Probabilistic example



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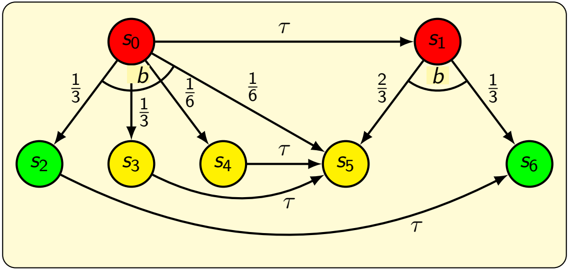

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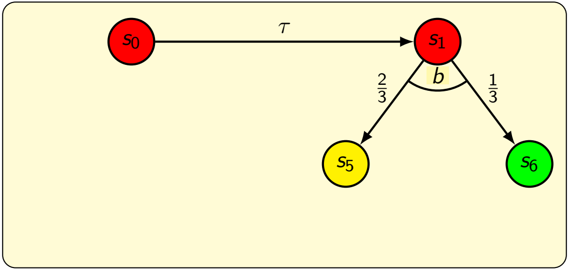

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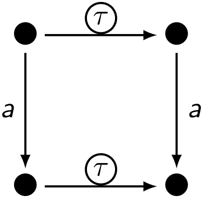

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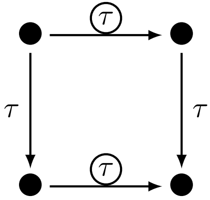

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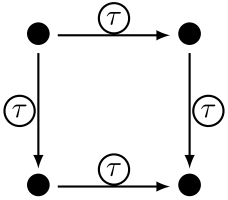

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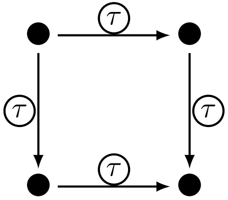

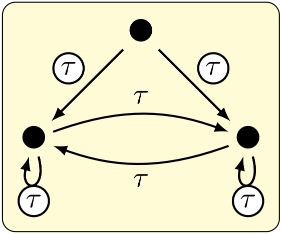

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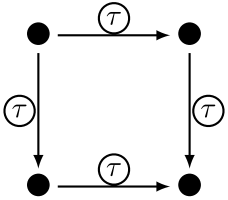

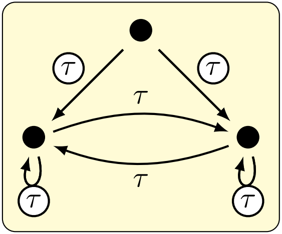

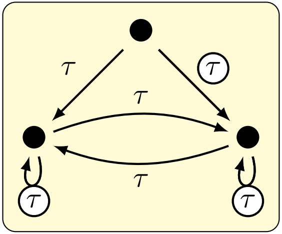

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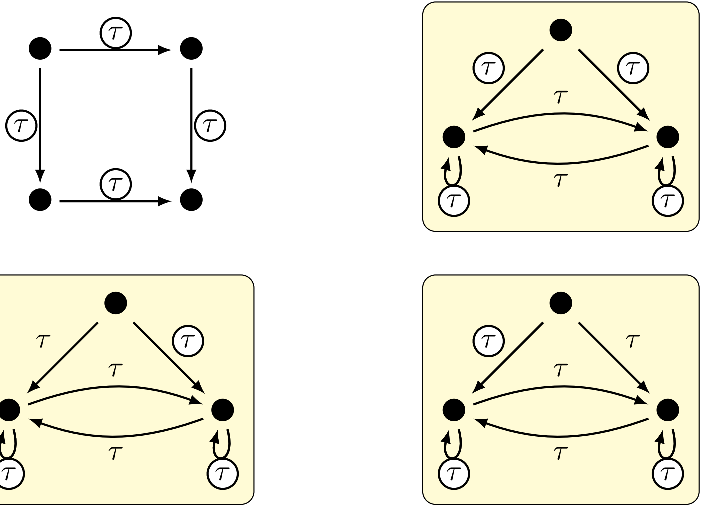

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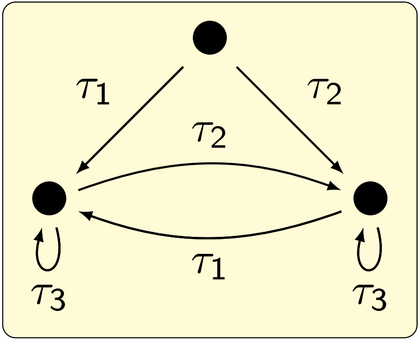

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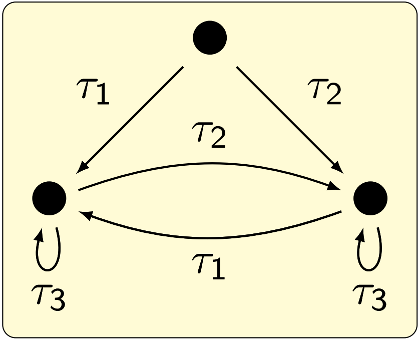

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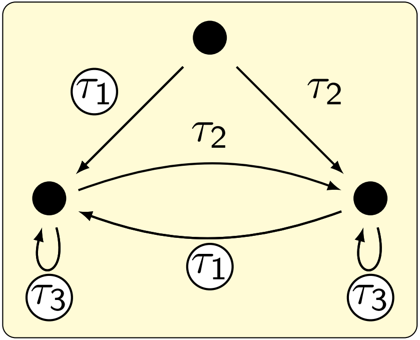

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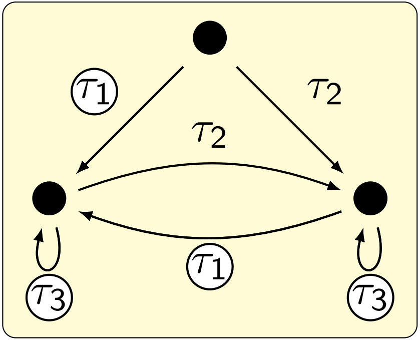

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Closure under unions is now really ensured.

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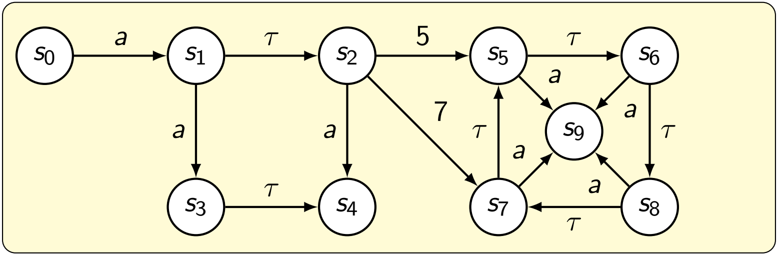

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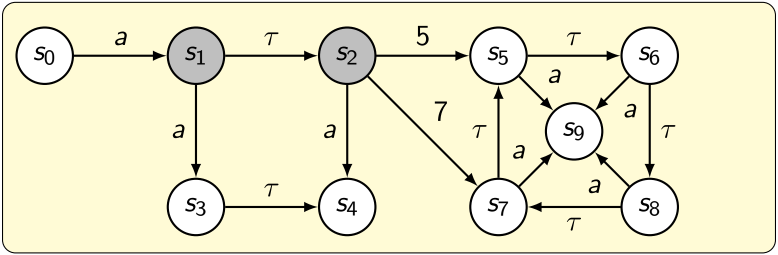

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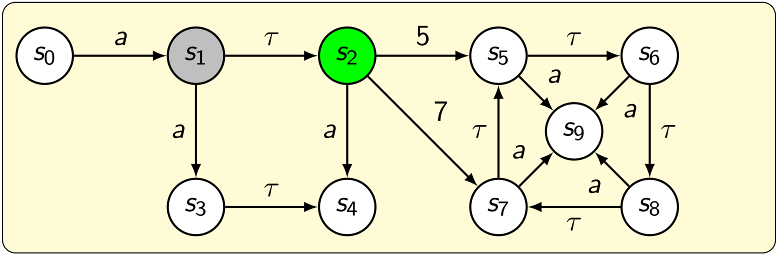

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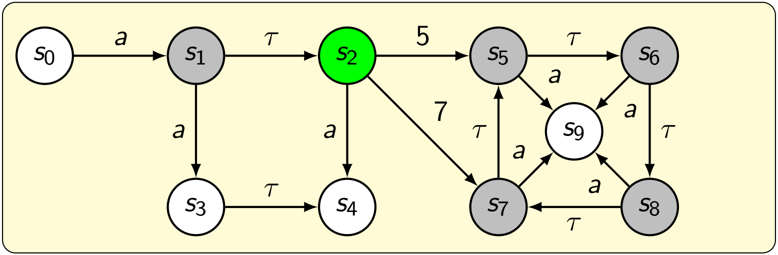

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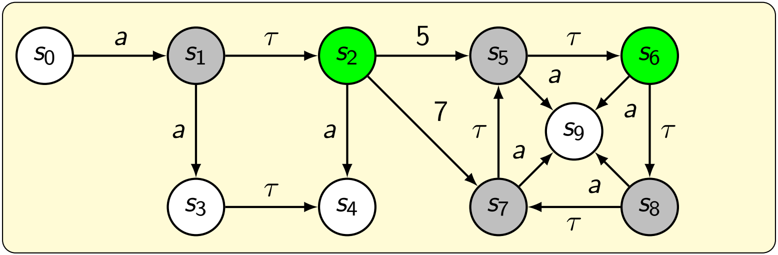

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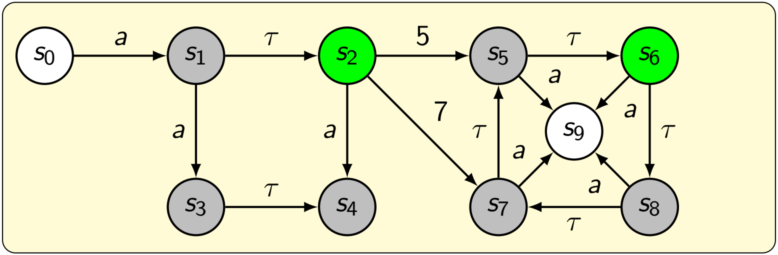

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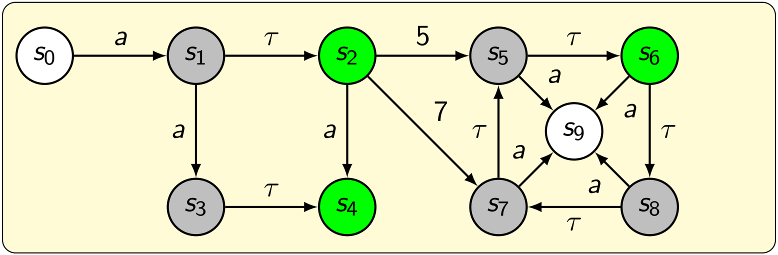

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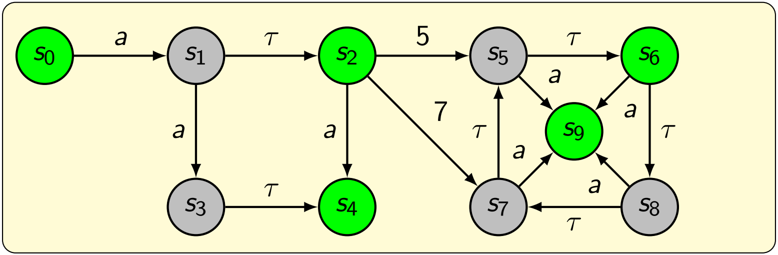

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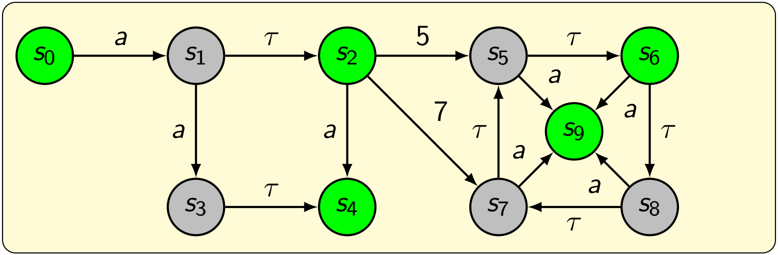

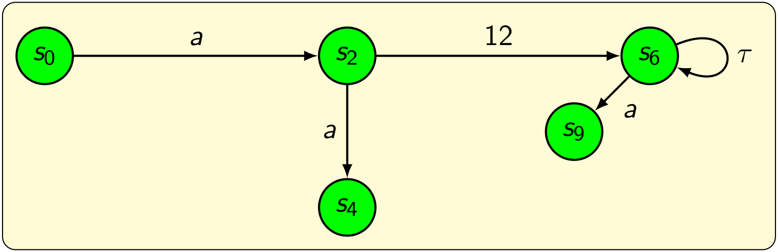

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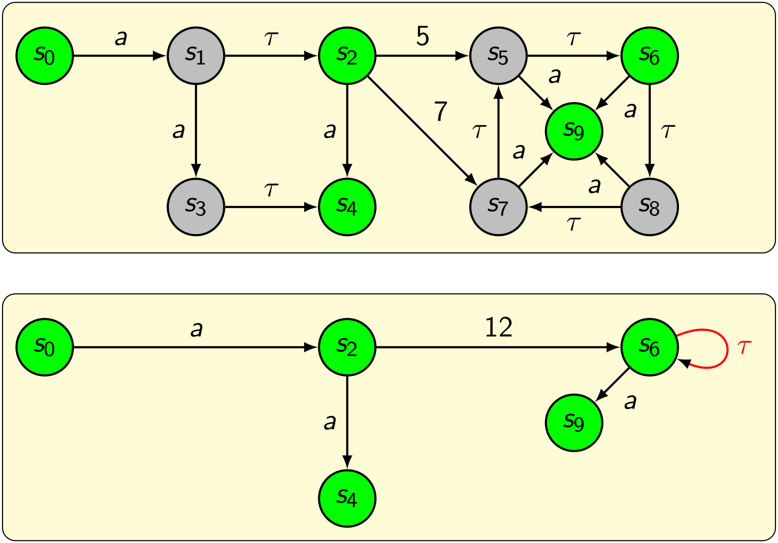

A process algebra for Markov automata: MAPA

Specification language MAPA:

- Based on μCRL (so data), with additional probabilistic choice and Markovian rates
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- Composibility via parallel composition, encapsulation, hiding and renaming

MLPPEs

We defined a special format for MAPA, the MLPPE:

$$X(\boldsymbol{g} : \boldsymbol{G}) = \sum_{i \in I} \sum_{\boldsymbol{d_i} : \boldsymbol{D_i}} c_i \Rightarrow a_i(\boldsymbol{b_i}) \sum_{\bullet \, \boldsymbol{e_i} : \boldsymbol{E_i}} f_i : X(\boldsymbol{n_i})$$

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Example of an MLPPE

GSPN-generated MAPA specification

$$
\begin{aligned}
&\mathit{System}(P_1 : \mathbb{N}, P_2 : \mathbb{N}, P_3 : \mathbb{N}, P_4 : \mathbb{N}, P_5 : \mathbb{N}, P_6 : \mathbb{N}) = \\
&\qquad P_1 \geq 1 \implies \tau \cdot \mathrm{System}(P_1 - 1, P_2 + 1, P_3, P_4, P_5, P_6) \\
&\quad + P_2 \geq 1 \implies \tau \cdot \mathrm{System}(P_1, P_2 - 1, P_3 + 1, P_4, P_5, P_6) \\
&\quad + P_5 \geq 1 \implies \lambda \cdot \mathrm{System}(P_1 + 1, P_2, P_3, P_4 + 1, P_5 - 1, P_6) \\
&\quad + P_6 \geq 1 \implies \mu \cdot \mathrm{System}(P_1, P_2, P_3, P_4 + 1, P_5, P_6 - 1) \\
&\quad + (P_3 \geq 1 \wedge P_4 \geq 1) \vee (P_4 \geq 1) \implies \tau \sum_{i : \{4,5\}} f : \\
&\qquad\quad \mathrm{System}(P_1, P_2, \text{if } i = 4 \text{ then } P_3 - 1 \text{ else } P_3, P_4 - 1, \\
&\qquad\qquad\quad \text{if } i = 4 \text{ then } P_5 + 1 \text{ else } P_5, \\
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Theorem

Every specification (without unguarded recursion) can be linearised to an MLPPE, preserving strong bisimulation.

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  - They does not disable each other
  - They should not influence each other's action
  - They should not influence each other's probability expression
  - Their order should not influence the next state

Heuristics for detecting confluence on MAPA

$$X(\boldsymbol{g} : \boldsymbol{G}) = \sum_{\boldsymbol{d_i} : \boldsymbol{D_i}} c_i \Rightarrow \tau \cdot X(\boldsymbol{n_i})$$
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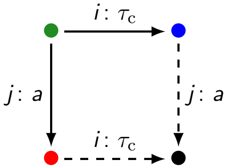

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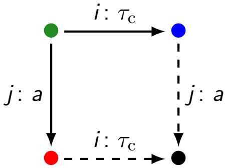

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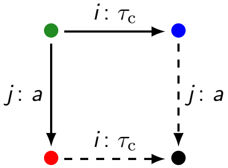

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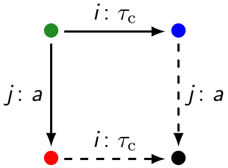

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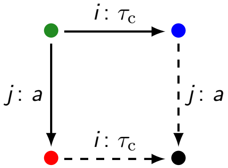

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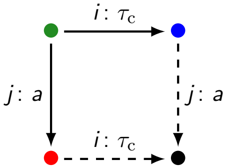

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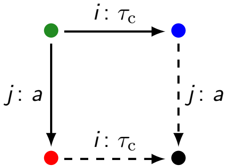

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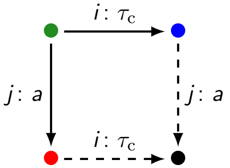

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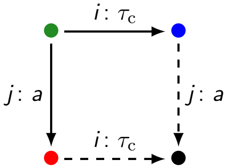

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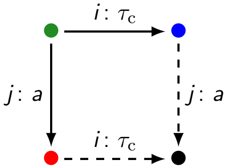

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We implemented:

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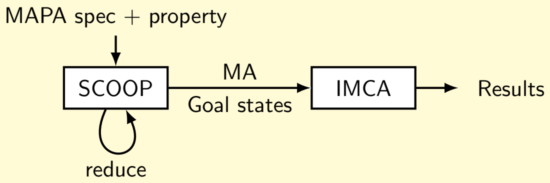

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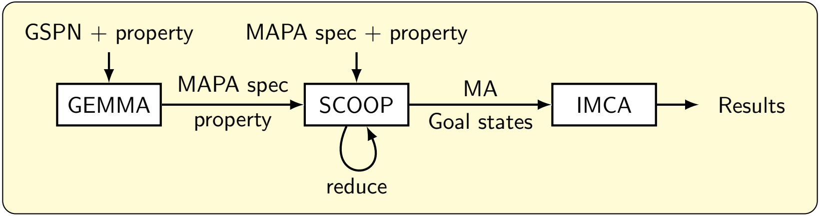

Case studies

Specification	Original state space			Reduced state space			Reduction	
	States	Trans.	IMCA	States	Trans.	IMCA	States	Time
`leader-3-7`	25,505	34,257	103.8	4,652	5,235	5.2	82%	90%
`leader-3-9`	52,465	71,034	214.3	9,058	10,149	9.9	83%	92%
`leader-3-11`	93,801	127,683	431.7	15,624	17,463	16.7	83%	93%
`leader-4-2`	8,467	11,600	74.9	2,071	2,650	5.2	76%	90%
`leader-4-3`	35,468	50,612	369.3	7,014	8,874	22.4	80%	92%
`leader-4-4`	101,261	148,024	1,325.3	17,885	22,724	62.2	82%	94%
`poll-2-2-4`	4,811	8,578	3.7	3,047	6,814	2.3	37%	32%
`poll-2-2-6`	27,651	51,098	90.9	16,557	40,004	49.1	40%	47%
`poll-2-4-2`	6,667	11,290	39.9	4,745	9,368	26.2	29%	32%
`poll-2-5-2`	27,659	47,130	1,573.8	19,721	39,192	1,053.5	29%	33%
`poll-3-2-2`	2,600	4,909	7.1	1,914	4,223	4.8	26%	29%
`poll-4-6-1`	15,439	29,506	330.0	4,802	18,869	109.3	69%	66%
`poll-5-4-1`	21,880	43,760	815.0	6,250	28,130	317.5	71%	61%
`grid-2`	2,508	4,608	2.8	1,393	2,922	1.1	44%	49%
`grid-3`	10,852	20,872	66.3	6,011	13,240	19.8	45%	67%
`grid-4`	31,832	62,356	922.5	17,565	39,558	316.5	45%	65%

Conclusions and future work

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- We introduced the first reduction technique for MAs abstracting from internal behaviour: confluence reduction
- It preserves divergences and is closed under unions
- We showed how to detect confluence on MAPA specifications and use the representation map approach to reduce on-the-fly
- Case studies show that significant reductions can be obtained

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Future work

- Develop even more powerful reduction techniques
- Define partial-order reduction as a restriction of confluence