Mathematical programming I, 2011:
Extra “short” exercises

Try to give a **short** and elegant proof. The extra exercises are compulsory and are to be handed in (latest) at the last WC (in groups of max 2 students).

Extra exercises due: Mo 9 May, before WC

Ex1. Let $M \in \mathbb{R}^{n\times n}$ be a nonsingular lower triangular matrix. Show that M^{-1} is also lower triangular.

Ex2. Let $A \in \mathbb{R}^{n\times n}$ be positive semidefinite and $x^T Ax = 0$. Show that $Ax = 0$ holds.

Ex3. Let $A \in \mathbb{R}^{n\times n}$ be positive definite. Show that A is also nonsingular.

Ex4. Show that any eigenvalue λ of a symmetric matrix $A \in \mathbb{R}^{n\times n}$ must be real. (Euclidian inner product in $\mathbb{C}^n$?)

Other exercises to be discussed in the first WC: Ex.2.11,2.13,2.18,2.26.

Extra exercises due: Mo 21 May, before WC

Ex5. (Ex.4.1) Let $C_i \subset \mathbb{R}^n$, $i \in I$ (I any index set) be closed, convex sets. Then the set $C = \cap_{i\in I} C_i$ is also closed, convex.

Ex6. Let $\| \cdot \|$ be a norm on $\mathbb{R}^n$ and let $y \in \mathbb{R}^n$ be fixed. Show that the function $f(x) = \|y - x\|$ is Lipschitz continuous on $\mathbb{R}^n$.

Other exercises to be discussed during the second WC: Ex.3.3;3.7;Ex.4.9, Ex.4.11;4.14;4.17;4.19.

Extra exercises due: Do 16 Juni, before WC

Ex7. Let be given $0 \neq d_0, d_2, \dots, d_{n-1} \in \mathbb{R}^n$ such that $d_j^T Ad_i = 0, \forall i \neq j$ ($A \succ 0$). Show that the vectors d_j are linearly independent.

Ex10. Show that A is positive definite if and only if A^{-1} is positive definite.

Ex11. (= Ex.5.20)

Other exercises to be discussed during the third WC: Ex.5.8;5.9;5.10;5.23. (in lecture sheets)