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What is Reachability Analysis?

- **Reachability Problem:** Given a graph $G = (V, E)$, initial states $I \subseteq V$ and goal states $F \subseteq V$, check if F is *reachable* from I via edges in E

- **In Model Checking:** Allows verification of most temporal safety properties *("something bad will never happen")*

- **Examples:**
  - Finding solutions in games (e.g. Chess, Sokoban, etc.)
  - Asserting mutual exclusion in parallel software
  - Asserting safety in traffic lights
  - etc.

State Space Explosions

- **Reachability Problem:** Given a graph $G = (V, E)$, initial states $I \subseteq V$ and goal states $F \subseteq V$, check if F is *reachable* from I via edges in E

- G is often *implicitly* described *(with a transition relation)*
- Set of reachable states is often determined *on-the-fly*
- Therefore, the size of G is often initially unknown

- **State Space Explosions:** occur when G does not longer fit into the available memory
  - Often happens in practice $\implies$ major limitation
  - State space Chess: $\sim 10^{43}$
  - Stars in universe: $\sim 10^{23}$

Fighting State Space Explosions

- **Reduction Techniques:**
  - Partial Order Reduction *(exploit commutative transitions)*
  - Bisimulation Minimization *(merge "similar" states)*
- **Compression Techniques:**
  - Decision Diagrams *(e.g. BDDs, MDDs, LDDs, ZDDs)*
  - SAT-based approaches *(e.g. IC3)*
- **Adding Hardware Resources:**
  - More memory $\Longrightarrow$ *larger* state spaces supported
  - More processors $\Longrightarrow$ *faster* reachability analysis

Parallel Symbolic Reachability

- **Symbolic Reachability:**
  - Represent the state space as a BDD
  - Represent initial states and the transition relation as BDDs
  - Perform reachability via BDD operations
- **Parallel Reachability:** Using a many-core cluster with a large amount of memory to perform reachability
  - Sylvan reaches speedups up to 38 with 48 cores
- **Disadvantages:**
  - Upgrading is *expensive*
  - Upgrading is *limited*

Distributed Symbolic Reachability

- **Distributed Reachability:** Using a network of workstations, connected via a high-performance network.

- **Compared to a Many-core Cluster:**
  - *Cheaper* scalability
  - *Unlimited* scalability

- **Challenges:**
  - Only small amounts of computation per memory access
  - Many *remote* memory accesses required
  - Network latency *easily* becomes a bottleneck

- **Achievements:** Very large state spaces are supported, but no speedups are obtained...

Improving Distributed Symbolic Reachability

- **Zhao et al (2009):** Most important design considerations for improvements are:
  - Data-distribution
  - Load-balancing maintenance
  - Reducing communication overhead
  - Exploting data-locality *(suggested by Chung et al)*

- **Contribution:** Employing *modern* techniques to implement these design considerations.

Motivation

- **Reducing Communicational Overhead:** Infiniband and RDMA
- **Load-balancing:** Work stealing *(due to the success of Sylvan and Lace)*
- **Exploting data-locality:** *Hierarchical* work stealing
- **Data Distribution:** RDMA-based distributed hash table

Research Questions

- **Main Question:** How efficient can RDMA-based distributed implementations of BDD operations scale along all processing units and available memory connected via a high-performance network?

- **Subquestions:**
  1. How can the storage and retrieval of data efficiently be managed to minimize their latencies?
  2. How can the total computational work be divided and distributed to maximize scalability along processors over a high-performance network?
  3. How can the idle-times of processes be minimized while performing network communication?

Research Method

- **Project:** We split the project into three parts:
  1. Storing states *(Distributed hash table)*
  2. Load-balancing mechanisms *(Hierarchical work stealing)*
  3. Distributed BDD operations

- All parts have separately been designed, implemented, and experimentally evaluated

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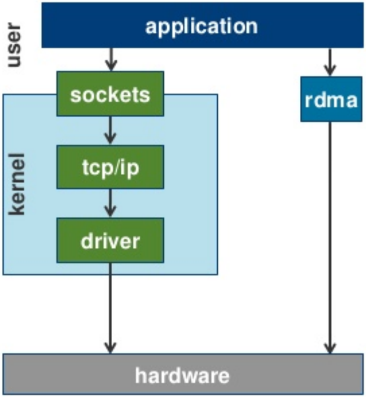
The Infiniband Architecture

- **Infiniband:** Specialized hardware used to construct high-performance networks
- **Advantages:**
  1. Comparable in price to standard Ethernet hardware
  2. Supports up to 100 GB/s
  3. NICs can *directly* access main-memory via PCI-E bus
  4. End-to-end latencies of $1\mu s$ have been measured *(according to the IB website)*
  5. Supports RDMA
- **UTwente:** 10 Dell M610 machines, connected via a 20 GB/s Infiniband network

Remote Direct Memory Access

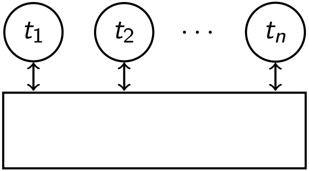
- **RDMA:** Directly access memory of a remote machine, without invoking its CPU
  - one-sided vs. two-sided RDMA
- **Advantages:**
  1. Zero-copy
  2. Kernel bypassing
  3. CPU efficiency
- **Roundtrip Latency:** Within $3\mu s$ in Infiniband hardware, compared to $60\mu s$ with TCP on Ethernet hardware

Kernel Bypassing



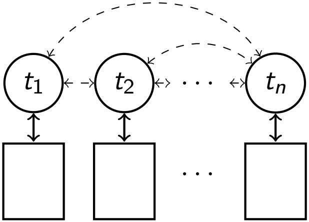
(VMWorld 2013 - How Latency Destroys Performance... And What to Do About It)

Parallel Programming Models



Shared Memory

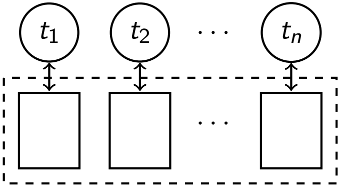
- Single addr. space
- e.g. NUMA, SMP

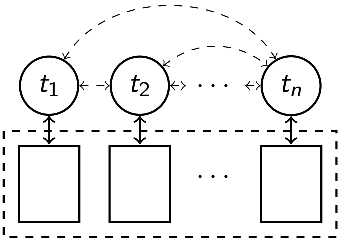


Distributed Memory

- Only local memories
- Communication via Message Passing

Partitioned Global Address Space





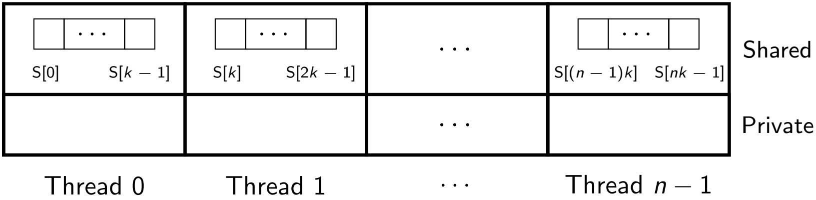
PGAS

- Shared + Distributed
- Data locality exploited

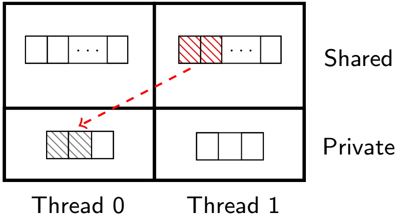
Hybrid PGAS

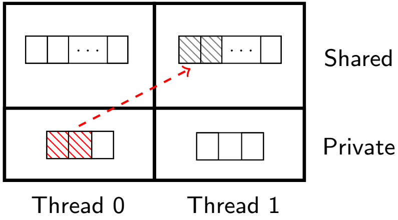
- PGAS + message passing

PGAS: Memory Model



PGAS: `memget` and `memput`





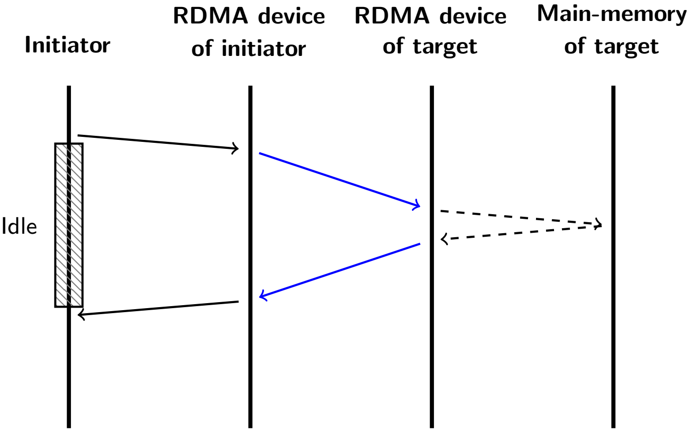
`memget`(P, S)

- Copies block of *shared* memory S into *private* memory P

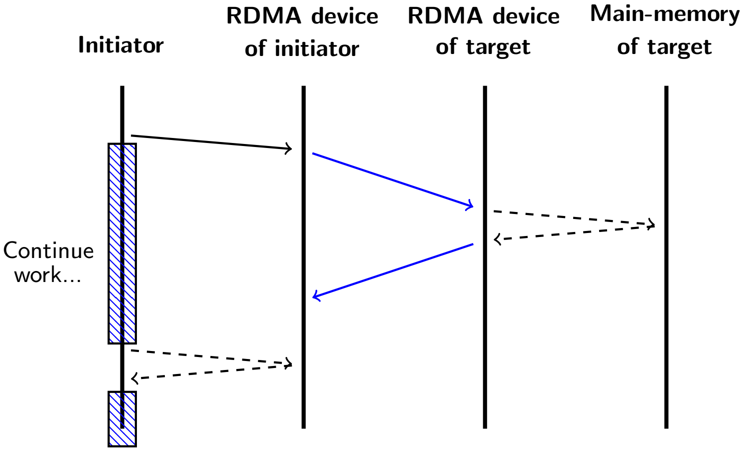
`memput`(S, P)

- Copies block of *private* P memory into *shared* memory S

PGAS: Synchronous Operations



PGAS: Asynchronous Operations

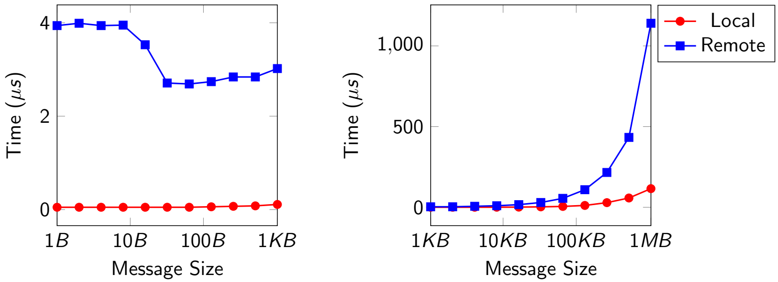


PGAS: Implementations

- **Many Implementations:**
  - Berkeley UPC
  - OpenSHMEM
  - Co-array Fortran
  - Titanium
  - X10
  - Chapel
  - etc.

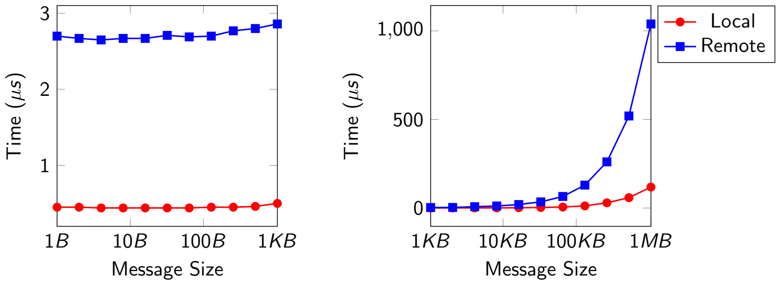
- We chose UPC because it supports async operations

PGAS: Latency of memget



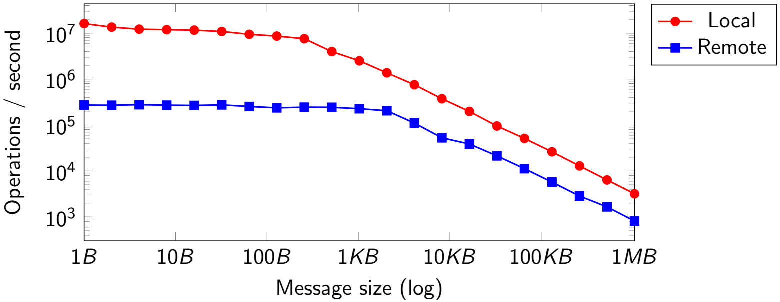
- **For messages ≤ 16 bytes:**
  - Local 76 times faster than remote
  - Local: 50ns on average
  - Remote: 3.87μs on average

PGAS: Latency of `memput`



- **For messages ≤ 16 bytes:**
  - Local only 6 times faster than remote
  - Local: $0.44\mu s$ on average
  - Remote: $2.68\mu s$ on average

PGAS: Throughput of `memput`



- **For messages ≤ 16 bytes:**
  - Local throughput 48 times higher than remote throughput

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Hash Table: Notations

Hash Table

$T = \langle b_0, \ldots, b_{n-1} \rangle$ as a sequence of buckets b_i, where:

- Hash table size: n, number of inserted elements: m
- Load factor: $\alpha = \frac{m}{n}$

Hash function

$h : U \to R$, with:

- Range of keys: $R = \{0, \ldots, r-1\}$
- Universe: $\{0,1\}^w$ *(of all w-sized binary words)*
- Mapping words $x \in U$ to buckets $b_{h(x)}$ by letting $r < n$

Notation

- For $x \in U$, we write $x \in b_i$ if bucket b_i contains x
- For $x \in U$, we write $x \in T$ if $x \in b_i$ for some $0 \leq i \leq n-1$

Distributed Hash Table: Requirements

Requirements

1. Minimal number of roundtrips
2. Minimal memory overhead
3. CPU efficient (no polling)
4. Should support `find-or-put`
5. Should support PGAS

`find-or-put`(d)

- If $d \in T$, return `found`
- If $d \notin T$, insert d and return `inserted`
- If $d \notin T$ and d cannot be inserted, return `full`

Resolving Collisions

Hash collision

Occurs when $h(x) = h(y)$ for $x, y \in U$ with $x \neq y$

- Hashing strategy determines the number of roundtrips required by `find-or-put`

Existing work (RDMA-based key/value stores)

- Pilaf *(Cuckoo hashing)*
- Nessie *(Cuckoo hashing)*
- FaRM *(Hopscotch hashing)*
- HERD *(high throughput, but CPU inefficient)*

We investigated hashing strategies and determined their performance

Chained Hashing

Chained Hashing

- Every bucket is a linked list
- Inserting $x \in U$ performed by adding it to $b_{h(x)}$
- Finding $x \in U$ performed by traversing $b_{h(x)}$

Complexity of `find-or-put`(d)

- $\Theta(m)$ in worst case *(when all m elements are in $b_{h(d)}$)*
- $\Theta(1 + \alpha)$ on average when a *universal hash function* is used

Universal hash function

$h : U \to R$ is called *universal* if $Pr[h(x) = h(y)] \leq \frac{1}{|U|}$ for every $x, y \in U$

- Good theoretical properties
- In practice: "cheaper" functions are often used

Cuckoo Hashing

Cuckoo Hashing

- Uses $k \geq 2$ *independent* hash functions $h_1, \ldots, h_k : U \rightarrow R$ with $h_i \neq h_j$ for every $i \neq j$ *(k-way Cuckoo hashing)*
- Nessie: 2-way Cuckoo hashing
- Pilaf: 3-way Cuckoo hashing

Cuckoo Invariant

For every element $x \in U$ it holds that either $x \notin T$ or $x \in b_{h_i(x)}$ for exactly one $1 \leq i \leq k$

Complexity

- Lookups require k roundtrips
- Inserts may require many when all k buckets are occupied

Bucketized Cuckoo Hashing

Bucketized Cuckoo Hashing

- Every bucket b_i is subdivided into l slots
- Every slot may contain an element from U
- Denoted by (k, l)-Cuckoo hashing

Bucketized Cuckoo Hashing

- Same as Cuckoo hashing, but linearly reduced by l
- Efficient even when $\alpha > 0.9$ *(Andersen et al, 2013)*
- Pilaf: $(2, 4)$-Cuckoo hashing could be very effective

Hopscotch Hashing

Hopscotch Hashing

- Each bucket b_i has a fixed-sized neighborhood $N(b_i)$ of constant size $H \geq 1$
- $N(b_i) = \langle b_i, \ldots, b_j \rangle$ with $j = (i + H - 1) \mod n$
- $N(b_i)$ thus contains b_i itself and the next $H - 1$ buckets *(modulo n)*
- Neighborhoods are thus consecutive in memory

Hopscotch Invariant

Let $x \in U$ and $N(b_{h(x)}) = \langle b_1, \ldots, b_H \rangle$. Then either $x \notin T$ or $x \in b_i$ for exactly one $1 \leq i \leq H$

Complexity

- `find-or-put`(d) may obtain $N(b_{h(d)})$ in 1 roundtrip
- Inserts may require *many* more when $N(b_{h(d)})$ is full

Linear Probing

Linear Probing

- For $x \in U$, it examines $b_{h(x)+0}, b_{h(x)+1}, \ldots, b_{h(x)+t}$ (modulo n) with threshold $t > 0$
- Buckets are consecutive in memory
- Therefore, cache-line efficient

Complexity

- Same as Hopscotch, but without relocation schemes
- Hopscotch invariant not maintained, lookups are more expensive
- But inserts are arguable cheaper *(amortized complexity)*

Complexity of Linear Probing

Theorem: Examining buckets *(Knuth, 1997)*

Assuming that a *universal hash function* is used, the *expected* number of buckets to examine until an empty bucket is found is at most:

$$\frac{1}{2}\Big(1 + \frac{1}{(1-\alpha)^2}\Big)$$

Chunk retrieval

- Similar to Hopscotch, a fixed-sized range of buckets can be obtained with a *single* roundtrip, which we refer to as *chunks*
- We denote the *chunk size* by $C \geq 1$

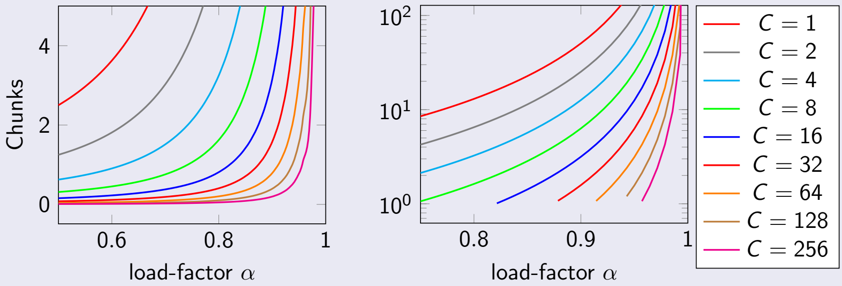
Complexity of Chunk Retrievals

Corollary: Number of chunks

The expected number of chunks to be inspected is at most:

$$\frac{1}{2C}\left(1 + \frac{1}{(1-\alpha)^2}\right)$$

Number of chunks to read



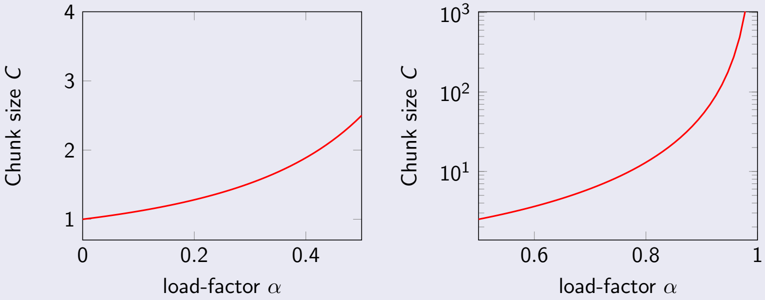
Bounding Efficiency

Theorem: Efficiency bound

A chunk of size $C \geq 1$ is expected to contain an empty bucket if:

$$\alpha \leq 1 - \sqrt{\frac{1}{2C - 1}}$$

Expected load-factor at which chunk is full

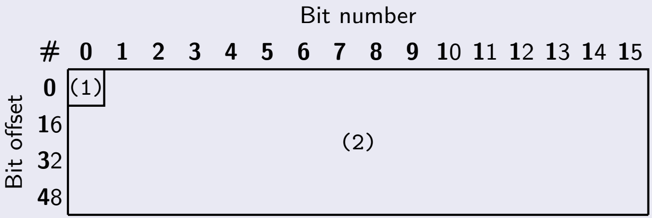


Designing `find-or-put`

Memory layout

- Shared array $B[0], \ldots, B[kn-1]$ of buckets, so that each thread *owns* k buckets
- 2D array $P[0][0], \ldots, P[M-1][C-1]$ per thread in *private* memory

Bucket layout



(1) is a locking bit (1 bit)

(2) contains data (63 bits)

Cache Efficiency

Cache lines

- Typically 64 bytes in size
- So 8 buckets per cache line
- Therefore, we choose C to be a multiple of 8

Cache line alignment

- The arrays P are cache line aligned
- The array B is *not*, since it is shared (could not find support from UPC to align shared memory)
- But the IB verbs libary *has* support for shared memory alignment...

Chunk Retrieval

Asynchronous chunk retrievals

- Before iterating over a chunk, request the *next* consecutive chunk
- Done to overlap roundtrips with actual work (interleaving queries)

The `query-chunk`(i, p) operation

- Transfers the ith chunk, starting from b_p, from B into $\mathrm{P}[i][0], \ldots, \mathrm{P}[i][C-1]$ *asynchronously*
- Returns a *handle* r for synchronization
- Synchronization can be performed by calling `sync`(r)

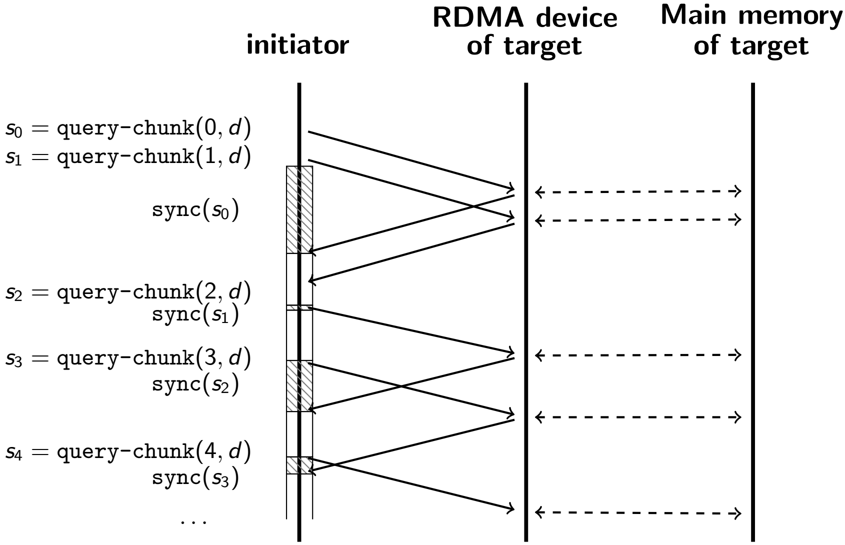
Design of `find-or-put`

```
def find-or-put(data):
    h ← hash(data)
    s_0 ← query-chunk(0, h)
    for i ← 0 to M − 1:
        if i < M − 1
            s_{i+1} ← query-chunk(i + 1, h)
        sync(s_i)
        for j ← 0 to C − 1:
            if (P[i][j] & OCCUPIED) = 0
                ▷ Try to claim bucket B[a]
                a ← (h + iC + j) mod kn
                d ← data(data) | OCCUPIED
                val ← cas(B[a], P[i][j], d)
                if val = P[i][j]
                    return inserted
                elif data(val) = data
                    return found
            elif data(P[i][j]) = data
                return found
    return full
```

```
def query-chunk(i, h):
    ▷ Find start and end index
    start ← (h + iC) mod kn
    end ← (h + (i + 1)C − 1) mod kn
    if end < start
        return split(start, end)
    else
        S ← ⟨B[start], ..., B[end]⟩
        P ← ⟨P[i][0], ..., P[i][C − 1]⟩
        return memget-async(S, P)
```

```
def split(start, end):
    ▷ Find the blocks in shared memory
    S_1 ← ⟨B[start], ..., B[kn − 1]⟩
    S_2 ← ⟨B[0], ..., B[end]⟩
    ▷ Corresp. blocks in private memory
    P_1 ← ⟨P[i][0], ..., P[i][|S_1| − 1]⟩
    P_2 ← ⟨P[i][|S_1|], ..., P[i][C − 1]⟩
    ▷ Retrieve the chunk
    s_1 ← memget-async(S_1, P_1)
    s_2 ← memget-async(S_2, P_2)
    return ⟨s_1, s_2⟩
```

Asynchronous Query Retrievals



Experimental Evaluation

Experimental setup (m610 partition)

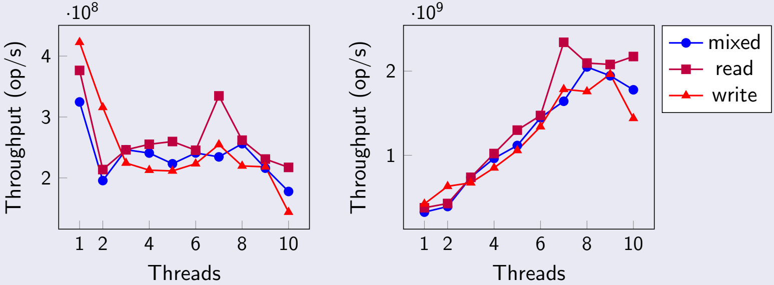
- 10 Dell M610 machines
- 8 GPU cores and 24 GB main-memory (each)
- Ubuntu 14.04.2 LTS, kernel version 3.13.0
- 20 GB/s Infiniband network

Benchmarks

- Throughput of `find-or-put`
- Latency of `find-or-put`
- Roundtrips required by `find-or-put`
- Under different workloads: mixed, read-intensive, and write-intensive

Local Throughput of `find-or-put`

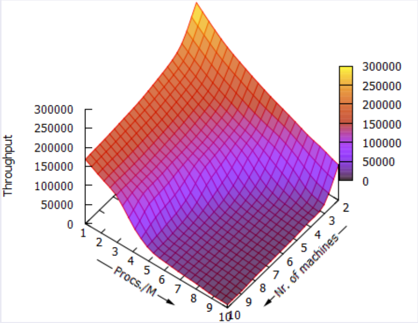
TP per thread (left) and total TP (right)

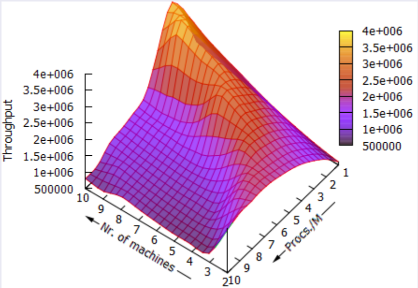


Workload	**Base Througput**		**Best Throughput**		**Speedup**
	Throughput	Procs.	Througput	Procs.	
Mixed	324,676,333	1	2,049,388,900	8	6.31
Read-intensive	376,434,000	1	2,342,278,333	7	6.22
Write-intensive	422,593,000	1	1,963,570,267	9	4.65

Remote Throughput of find-or-put

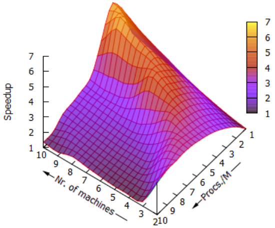
Mixed workload: TP per thread (left) and total TP (right)



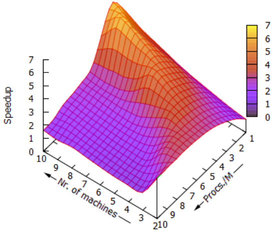


Workload	Base Througput			Best Throughput			Speedup
	TP.	M.	Procs./M.	TP.	M.	Procs./M.	
Mixed	592,929	2	1	3,607,003	10	3	6.08
Read	742,728	2	1	4,620,752	10	3	6.22
Write	495,370	2	1	2,999,234	10	3	6.05

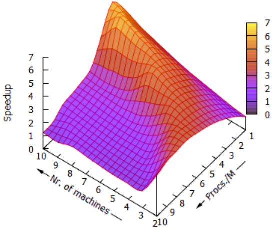
Speedups in Remote Throughput



Mixed



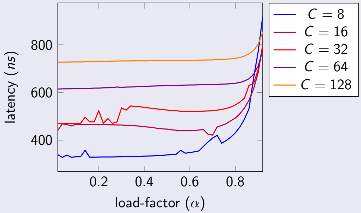
Read-intensive



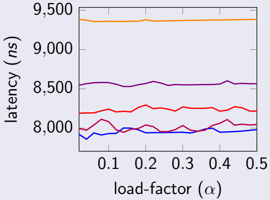
Write-intensive

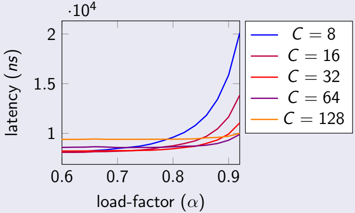
Latency of `find-or-put`

Local latency



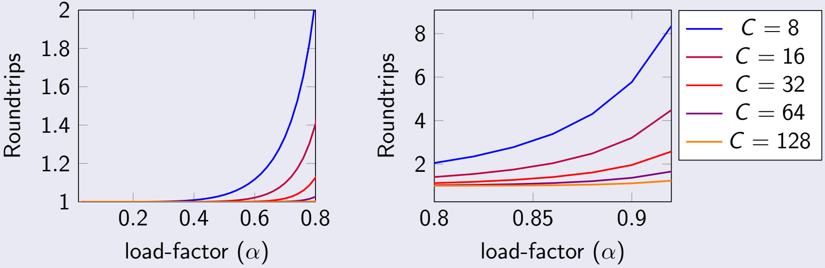
Remote latency





Roundtrips Required by `find-or-put`

Number of roundtrips



Conclusions

General

- Minimizing roundtrips *critical* for increased performance
- Overlapping queries reduces waiting-times and increases latency
- Linear probing requires less roundtrips than Hopscotch and Cuckoo

Performance

- `find-or-put` takes $9.3\mu s$ on average with $\alpha = 0.9$ and $C = 64$
- Peak-throughput of 3.6×10^6 op/s obtained with 10 machines

Future work

- Use adaptive chunk sizes (based on efficiency bounds Theorem)
- In addition, update asynchronous queries to prevent unused retrievals

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Load Balancing

Task-based parallelism

- Dividing computational problems into smaller *tasks*
- Task is a basic unit of work and only depend on intermediate *subtasks*
- All threads maintain *task pools*

Load-balancing tasks

- Ideally tasks are perfectly distributed *(infeasible)*
- Instead: mapping tasks dynamically to threads

Task granularity

The relation between the computational workload and the amount of communication required between threads

- Fine-grained: large number of small tasks
- coarse-grained: small number of large tasks

Work Stealing and Sharing

Work stealing

- Efficient technique for fine-grained task parallelism
- Threads are either idle or working
- When idle, threads *steal* from remote task pools
- Stealing thread is *thief*, targetted thread is *victim*
- Termination when all threads are idle

Work sharing

- Threads communicate their status
- When idle, other threads *share* work
- Communication of work stealing is *more* efficient (Blumofe, 1999)

Work Stealing Operations

Operations

- **spawn**: push new task to task pool
- **call**: execute given task
- **sync**: pull task from pool and execute

Fibonacci example

```
int fib(n):
    if n < 2 return n
    a ← fib(n − 1)
    b ← fib(n − 2)
    return a + b
```

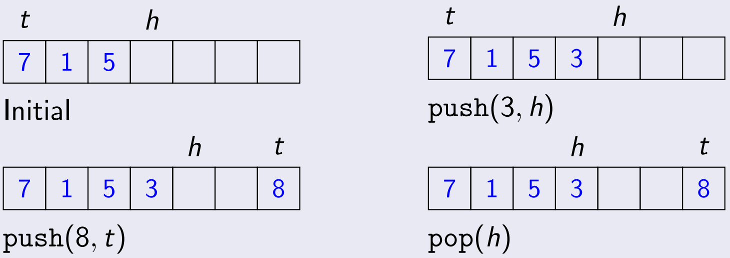
```
int par-fib(n):
    if n < 2 return n
    spawn(par-fib, n − 1)
    r ← call(par-fib, n − 2)
    return r + sync
```

Implementing the Task Pool

Double ended queue (deque)

- Similar to queue, but has two ends: *head* and *tail*
- Items can be pushed or popped from both ends
- Implemented as a fixed-sized array

Example

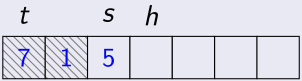


Implementing the Task Pool

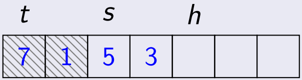
Split deque

- Deque with a *split point* s
- s determines what sections belongs to *head* and *tail*
- Used to denote a *public* and *private* region
- s can be relocated to increase/decrease public region

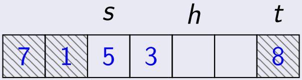
Example



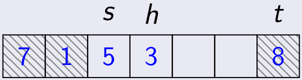
Initial



push$(3, h)$



push$(8, t)$



pop(h)

Implementing the Task Pool

Performance of split deques

- Modifying *s* may conflict with steal operations
- Either locks or memory fences required
- Expensive in distributed environment!

Existing work (current state-of-the-art)

- HotSLAW: access to public region requires locking
- Scioto: *whole* split deque locked when stealing
- Lace: non-blocking, but shrinking public region requires memory fence

Implementing the Task Pool

Private deques

- Implemented as a stack
- Do not have a public region (completely private)

Private deque work stealing

- When stealing, idle workers *explicitly* ask for work
- **Advantage:** No locking required
- **Disadvantage:** Requires participation from both victim and thief

Victim Selection Protocols

Selecting victims

- Random victim selection
- Hierarchical victim selection (Min et al, 2011)
- Leapfrogging

Contribution and motivation

Private-deque work stealing operations:

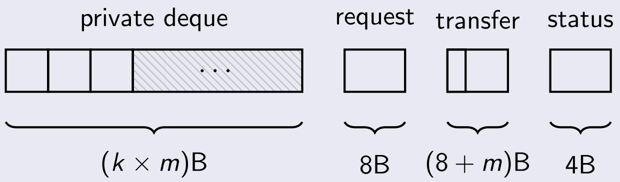
- Minimal number of roundtrips
- Uses all three victim selection protocols
- Similar approach by Olivier et al. (2008), but requires *more* roundtrips and does *not* exploit network hierarcy

Designing Private-Deque Work Stealing

Memory layout

- Shared 2D-array: deque[0][0], . . . , deque[THREADS − 1][k − 1]
- Request cells: request[0], . . . , deque[THREADS − 1]
- Transfer cells: transfer[0], . . . , transfer[THREADS − 1]
- Status cells: status[0], . . . , status[THREADS − 1]

Schematically



Where m is the task size (in Bytes) and k the deque size

Performing Steals

Request cell

- Contains either BLOCKING, EMPTY, or a thread ID:
- **blocking**: no tasks can be stolen
- **empty**: no pending steal requests
- **identifier**: pending steal request

Transfer cell

- Contains either EMPTY or a task + location:
- **empty**: no task received
- **task**: task received + corresponding location in deque

Status cell

- Contains either IDLE or WORKING

Hierarchical Work Stealing

Domain levels

- Berkeley UPC provides `thread-distance`(i, j) function
- Which returns: verynear, near, far, or veryfar
- We use an array *domain*, so that *domain*$[i]$ contain all thread IDs on the ith level
- We use a `shuffle` function that randomly *shuffles* a domain level.

Hierarchical stealing

1. Threads start by performing leapfrogging
2. Threads perform `count`(*domain*$[i]$) steal attempts before moving to level $i + 1$
3. If all levels have been tried, perform termination detection

Designing **spawn**, **call**, and **sync**

```
1  def sync():
2      task ← deque[MY-ID][head − 1]
3      if task.stolen
4          communicate()
5          while ¬task.completed:
6              ▷ Perform leapfrogging
7              if steal(task.owner) continue
8              if task.completed break
9              ▷ Perform hierarchical stealing
10             for i ← 0 to HIERARCHY-LVLS - 1:
11                 shuffle(domain[i])
12                 foreach victim ∈ domain[i] do
13                     if steal(victim) goto line 5
14                     if task.completed goto line 16
15         ▷ Return result from stolen task
16         head ← head − 1
17         tail ← tail − 1
18         return task.result
19     else
20         head ← head − 1
21         return call(task)
```

```
def spawn(desc, params):
    ▷ Build a new task
    task ← deque[MY-ID][head]
    task.desc ← desc
    task.stolen ← false
    task.completed ← false
    task.params ← params
    ▷ Write new task to deque
    deque[MY-ID][head] ← task
    head ← head + 1
```

```
def call(desc, params):
    communicate()
    ▷ Find the intended function
    func ← function-of(desc)
    ▷ Invoke that function
    return func(params)
```

Initiating a Computation

```
1  def initiate(desc, params):
2      ▷ Wait for all workers to start
3      status[MY-ID] ← WORKING
4      barrier()
5      ▷ Perform task
6      result ← call(desc, params)
7      ▷ Wait for all workers to complete
8      status[MY-ID] ← IDLE
9      barrier()
10     return result
```

```
1  def compute(desc, params):
2      if MY-ID = 0
3          initiate(desc, params)
4      else
5          participate()
```

```
1  def participate():
2      ▷ Wait for all workers to start
3      status[MY-ID] ← IDLE
4      barrier()
5      ▷ Perform hierarchical stealing
6      while true:
7          status[MY-ID] ← IDLE
8          for i ← 0 to HIERARCHY-LVLS - 1:
9              shuffle(domain[i])
10             foreach victim ∈ domain[i] do
11                 if steal(victim) goto line 5
12         ▷ No worker had tasks to steal..
13         if termination-detection()
14             break
15     barrier()
```

Communicate Work

```
def communicate():
    if head − tail < 2
        if request[MY-ID] ≠ BLOCKED
            ▷ Not enough stealable tasks, block further requests
            if request[MY-ID] ≠ EMPTY
                reject-and-block()
            elif cas(request[MY-ID], EMPTY, BLOCKED) ≠ EMPTY
                reject-and-block()
    elif request[MY-ID] = BLOCKED
        request[MY-ID] ← EMPTY
    elif request[MY-ID] ≠ EMPTY
        thief ← request[MY-ID]
        request[MY-ID] ← EMPTY
        ▷ Prepare task to be stolen
        deque[MY-ID][tail].stolen ← true
        deque[MY-ID][tail].owner ← thief
        ▷ Construct the transfer message
        msg ← new TransferMessage
        msg.index ← tail
        msg.task ← deque[MY-ID][tail]
        memput-async(transfer[thief], msg)
```

Stealing and Termination Detection

```
def steal(victim):
    communicate()
    transfer[MY-ID] ← EMPTY
    res ← cas(request[victim], EMPTY, MY-ID)
    if res = EMPTY
        ▷ Wait for response from victim
        while transfer[MY-ID] = EMPTY:
            communicate()
        if transfer[MY-ID] = EMPTY
            return false
        else
            status[MY-ID] ← WORKING
            i ← transfer[MY-ID].index
            task ← transfer[MY-ID].task
            task.result ← call(task)
            ▷ Write back the task result
            task.completed ← true
            memput-async(deque[victim][i], task)
            return true
    return false
```

```
def reject-and-block():
    ▷ Block further requests
    thief ← request[MY-ID]
    request[MY-ID] ← BLOCKED
    ▷ Send a negative response
    msg ← new TransferMessage
    msg.index ← 0
    msg.task ← EMPTY
    memput-async(transfer[thief], msg)
```

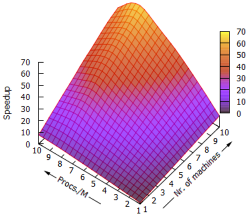
```
def termination-detection():
    ▷ Send status requests
    for i ← 0 to THREADS − 1:
        s_i ← memget-async(
            res[i], status[i])
    ▷ Wait for status responses
    for i ← 0 to THREADS − 1:
        sync(s_i)
        if res[i] = WORKING
            return false
    return true
```

Benchmarks

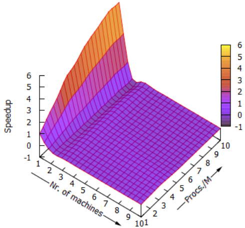
- Performed a number of microbenchmarks
- Determined speedup when scaling along machines and threads per machine
- Compared speedup with HotSLAW

Benchmark	**Nr. of Tasks**	**Avg. Task Time**	**Input/Output Size**	**Input/Output Size Hotslaw**
`fib`(45)	3,672,623,805	0.154 μs	16/8 bytes	4/8 bytes
`nqueens`(15)	171,127,071	4.14 μs	20/8 bytes	28/8 bytes
`uts(T2L)`	96,793,510	0.986 μs	20/8 bytes	32/0 bytes
`uts(T3L)`	111,345,631	0.722 μs	20/8 bytes	32/0 bytes
`matmul`(512)	32,767	188.30 μs	20/8 bytes	28/0 bytes

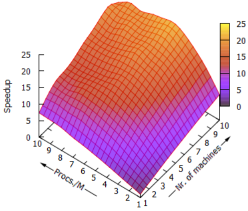
Speedup Graphs



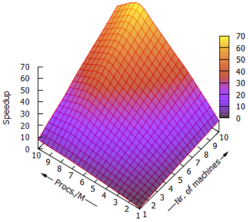
fib(45)



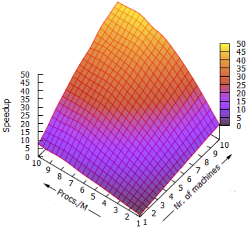
matmul(512)



uts(T3L)



nqueens(15)



uts(T2L)

Computation Times

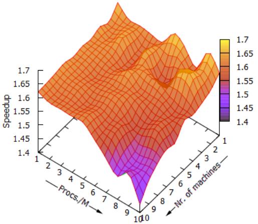
Sequential- and best times

Benchmark	Sequential Time	Best Configuration Time	Best Configuration Machines	Best Configuration Procs./M.	Speedup
`fib(45)`	563.87	8.85	10	8	63.69
`matmul(512)`	6.17	1.07	1	9	5.76
`uts(T2L)`	90.48	1.90	10	8	47.62
`uts(T3L)`	73.60	3.48	10	5	21.15
`nqueens(15)`	707.64	10.29	10	8	68.74

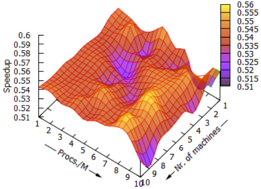
Comparison with HotSLAW

Benchmark	Our Implementation Seq. Time	Our Implementation Best Time	HotSLAW Seq. Time	HotSLAW Best Time
`fib(45)`	563.87	8.85	938.49	13.13
`nqueens(15)`	707.64	10.29	387.53	5.68
`uts(T2L)`	90.48	1.90	81.22	1.53
`uts(T3L)`	73.60	3.48	67.64	5.48
`uts(T3L)`*	73.60	3.48	51.69	1.32

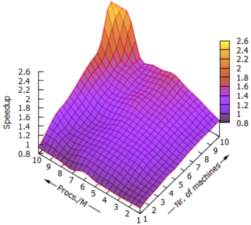
Comparison with HotSLAW (Computation Time)



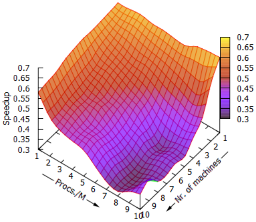
fib(45)



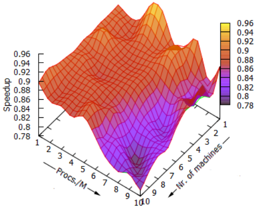
nqueens(15)



uts(T3L)

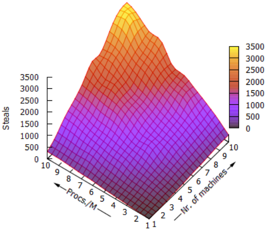


uts(T3L) with steal-10

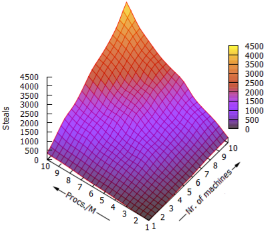


uts(T2L)

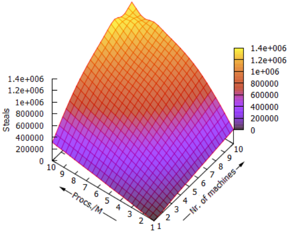
Average Number of Steals



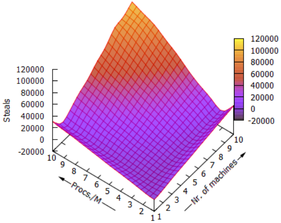
fib(45)



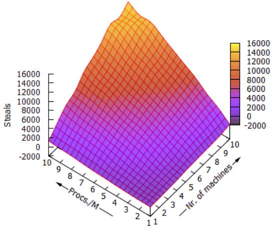
nqueens(15)



uts(T3L)

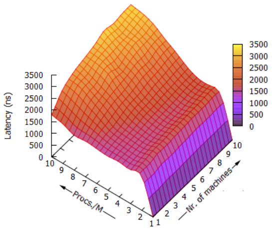


uts(T3L) with steal-10

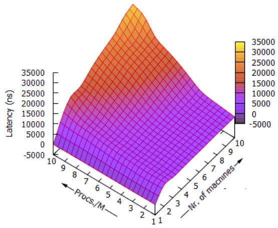


uts(T2L)

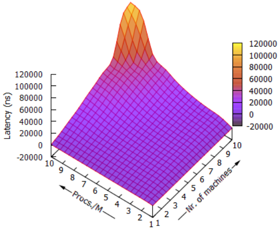
Latency of steal



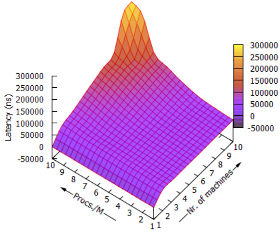
local



remote

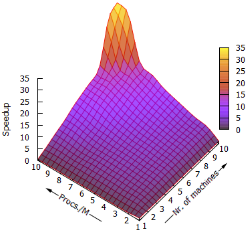


HotSLAW - local

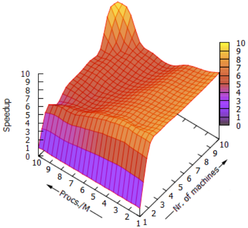


HotSLAW - remote

Speedup of steal (vs HotSLAW)



local



remote