

# Verifying Concurrent Software

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# Concurrent programming is error-prone



# Program Logics (Tony Hoare)



# Hoare Triple

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# Hoare Triple

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Formal meaning:

$$\forall \sigma : \text{STORE}, \sigma \models P \wedge \langle S, \sigma \rangle \rightsquigarrow^* \langle \text{done}, \sigma' \rangle \rightarrow \sigma' \models Q$$

# Hoare Logic

$$\overline{\{P\} \text{ SKIP } \{P\}}$$

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$$\frac{}{\{P\} \text{SKIP} \{P\}} \qquad \frac{\{P\} S_1 \{R\} \quad \{R\} S_2 \{Q\}}{\{P\} S_1; S_2 \{Q\}}$$

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$$\frac{\{P \wedge B\} S_1 \{Q\} \quad \{P \wedge \neg B\} S_2 \{Q\}}{\{P\} \text{IF } B \text{ THEN } S_1 \text{ ELSE } S_2 \{Q\}}$$

# Hoare Logic

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$$\frac{\{P \wedge B\} S_1 \{Q\} \quad \{P \wedge \neg B\} S_2 \{Q\}}{\{P\} \text{IF } B \text{ THEN } S_1 \text{ ELSE } S_2 \{Q\}}$$

$$\frac{\{I \wedge B\} S \{I\}}{\{I\} \text{WHILE } B \text{ DO } S \{I \wedge \neg B\}}$$

# Program Specifications (Rustan Leino)

```
void method( $\dots$ ) {  
     $\dots$   
}
```

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```
requires precondition  
void method( $\dots$ ) {  
     $\dots$   
}
```

# Program Specifications (Rustan Leino)

```
requires precondition  
ensures postcondition  
void method( $\dots$ ) {  
     $\dots$   
}
```



# Specification Example: Sorting

```
void sort(int[] a) {  
    ...  
}
```

# Specification Example: Sorting

```
requires  $a \neq \text{null}$   
void sort(int[] a) {  
    ...  
}
```

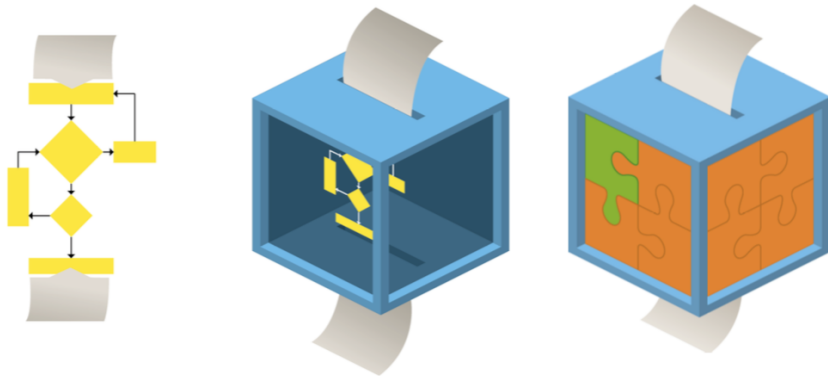
# Specification Example: Sorting

**requires**  $a \neq \text{null}$

**ensures**  $\forall i : \text{INT}, (0 \leq i < a.\text{length} - 1) \rightarrow a[i] \leq a[i + 1]$

```
void sort(int[] a) {  
    ...  
}
```

# Static Verifiers



# Separation Logic (John Reynolds)



# Separation Logic: Pointers

```
class Counter {  
    int count;  
  
    public void incr(int x) {  
        count = count + x;  
    }  
}
```

# Separation Logic: Pointers

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class Counter {  
    int count;  
  
    requires this.count  $\rightarrow ?v$   
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# Separation Logic: Pointers

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class Counter {  
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## Separation Logic: Separating Conjunction

$$h, \sigma \models P_1 * P_2$$

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## Separation Logic: Separating Conjunction

$$h, \sigma \models P_1 * P_2$$

if and only if

$$h = h_1 \uplus h_2, \text{ such that } h_1, \sigma \models P_1 \text{ and } h_2, \sigma \models P_2$$

# Concurrent Separation Logic (Peter O'Hearn)

$$\overline{\{P_1 * P_2\} S_1 \parallel S_2 \{Q_1 * Q_2\}}$$

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$$\frac{\{P_1\} S_1 \{Q_1\} \quad \{P_2\} S_2 \{Q_2\}}{\{P_1 * P_2\} S_1 \parallel S_2 \{Q_1 * Q_2\}}$$

# Separation Logic with Permissions (Boyland)

---

$$\{c.val \xrightarrow{1} ?v\} x = c.val + 1 \parallel y := c.val + 2 \{c.val \xrightarrow{1} ?v\}$$



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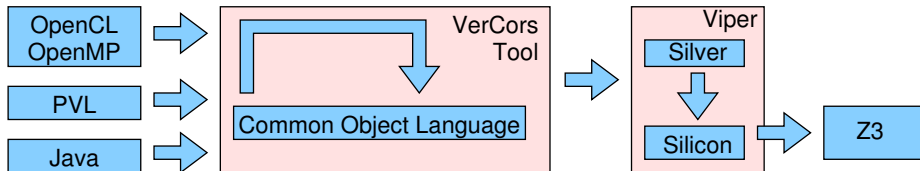
# Separation Logic with Permissions (Boyland)

$$\frac{\begin{array}{l} \{c.val \xrightarrow{\frac{1}{2}} ?v\} x = c.val + 1 \{c.val \xrightarrow{\frac{1}{2}} ?v\} \\ \{c.val \xrightarrow{\frac{1}{2}} ?v\} y = c.val + 2 \{c.val \xrightarrow{\frac{1}{2}} ?v\} \end{array}}{\{c.val \xrightarrow{1} ?v\} x = c.val + 1 \parallel y := c.val + 2 \{c.val \xrightarrow{1} ?v\}}$$

# The VerCors Toolset



# The VerCors Toolset: Overview



## Example: Verifying Loop Parallelisations

---

```
for(int  $i = 0; i < N; i ++$ ) $\{S_i\}$ 
```

## Example: Verifying Loop Parallelisations

---

$\{*\}_{j=0}^{N-1} P_j$  **for**(int  $i = 0; i < N; i ++$ ) $\{S_i\}$   $\{*\}_{j=0}^{N-1} Q_j$

## Example: Verifying Loop Parallelisations

$$\frac{\{P_0\} S_0 \{Q_0\} \quad \cdots \quad \{P_{N-1}\} S_{N-1} \{Q_{N-1}\}}{\{*\}_{j=0}^{N-1} P_j \text{ for}(\text{int } i = 0; i < N; i++)\{S_i\} \{*\}_{j=0}^{N-1} Q_j}$$

## Example: Verifying GPGPU Kernels

```
kernel void add(global int A, global int B) {  
    A[tid] = 3 * B[tid];  
}
```



## Example: Verifying GPGPU Kernels

**requires**  $A[tid] \xrightarrow{1} - * B[tid] \xrightarrow{\frac{1}{2}} -$   
**kernel void** *add*(**global int** *A*, **global int** *B*) {  
     $A[tid] = 3 * B[tid];$   
}

## Example: Verifying GPGPU Kernels

**requires**  $A[tid] \xrightarrow{1} - * B[tid] \xrightarrow{\frac{1}{2}} -$   
**ensures**  $A[tid] \xrightarrow{1} 3B[tid] * B[tid] \xrightarrow{\frac{1}{2}} -$   
**kernel void** *add*(**global int** *A*, **global int** *B*) {  
     $A[tid] = 3 * B[tid];$   
}

# Example: Model-based Verification

**Program**  
permission-based  
separation logic

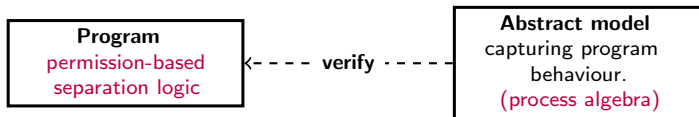
# Example: Model-based Verification

## Program

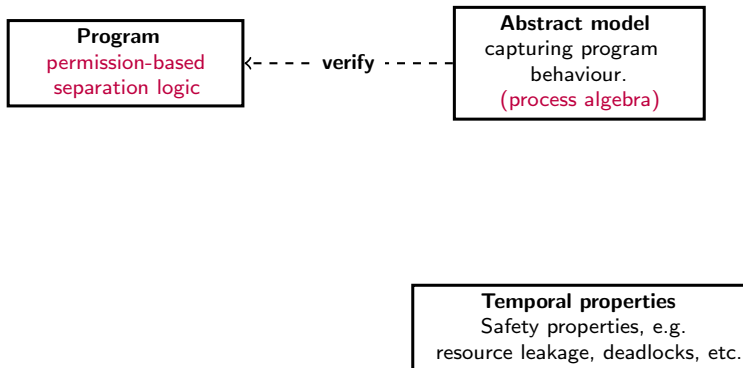
permission-based  
separation logic

**Abstract model**  
capturing program  
behaviour.  
(process algebra)

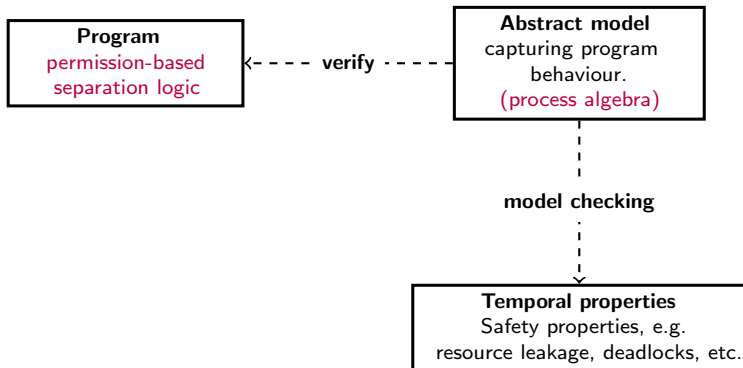
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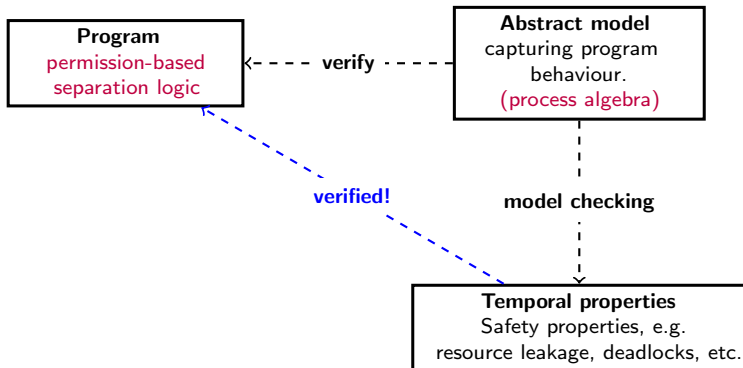
# Example: Model-based Verification



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# Example: Model-based Verification





# Model-based verification: future directions

```
requires ...  
requires Process(P(k) · Q)  
ensures ...  
ensures Process(Q)  
void main(int k):  
    int v ← MPI_Recv(*)  
    MPI_Send(0, v + k)
```

# Model-based verification: future directions

```
requires . . .
```

```
requires Process(P(k) · Q)
```

```
ensures . . .
```

```
ensures Process(Q)
```

```
void main(int k):
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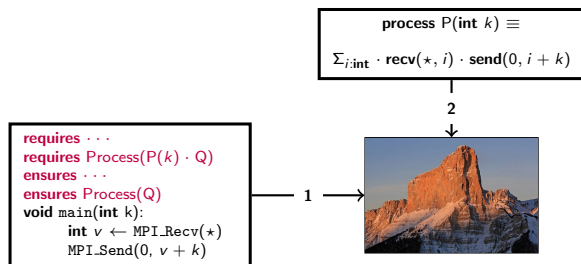
```
    int v ← MPI_Recv(★)
```

```
    MPI_Send(0, v + k)
```

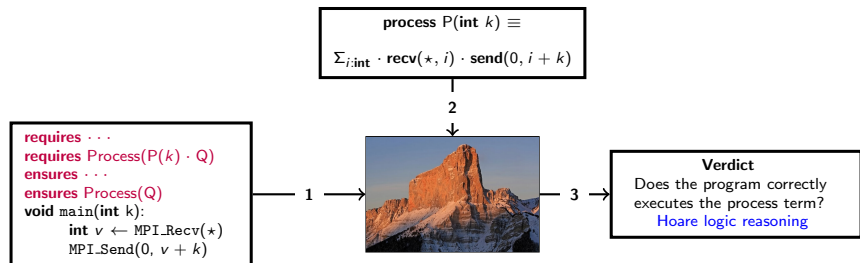
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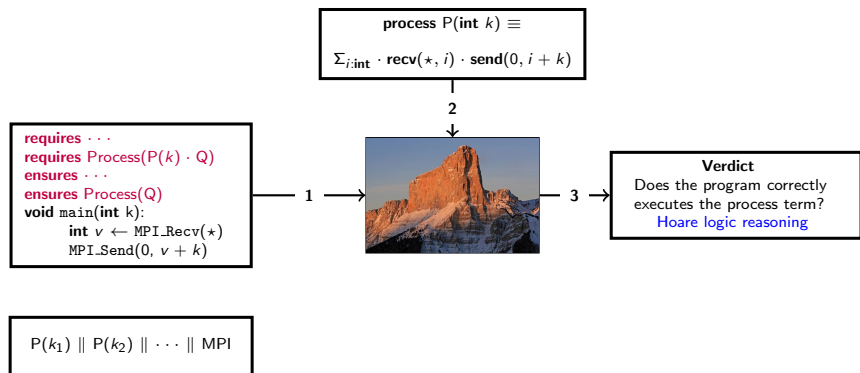
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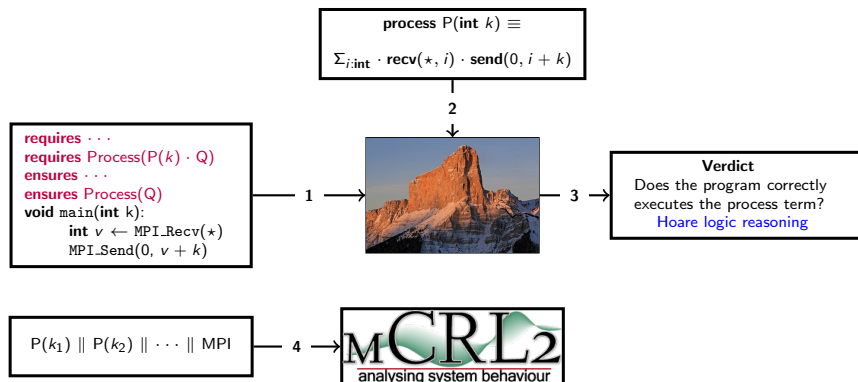
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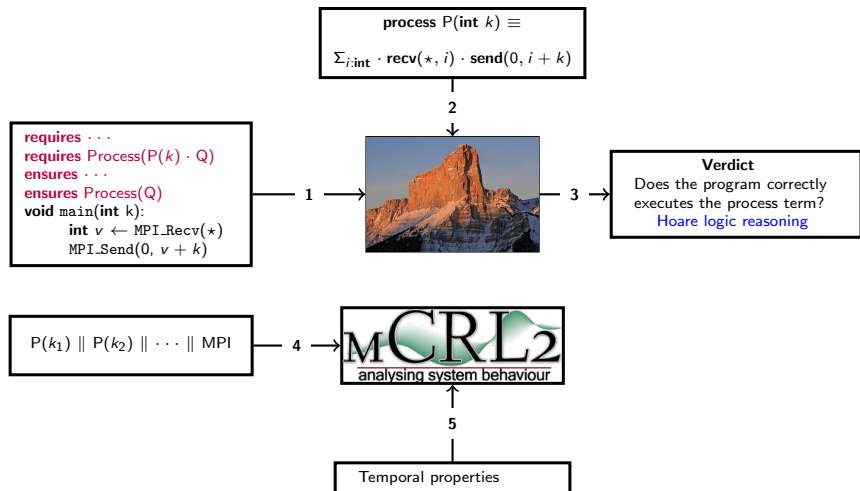
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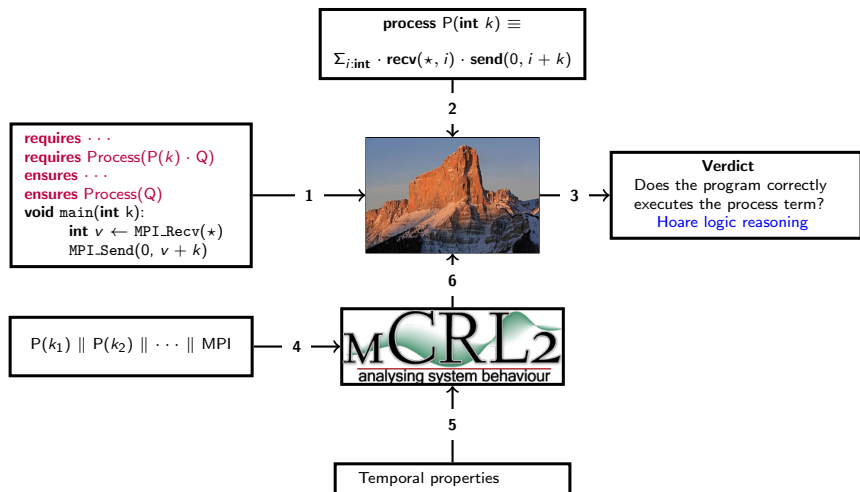
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